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Asymptotic behavior of semi-quasivariational optimistic bilevel problems in Banach spaces

M. Beatrice Lignola [a], Jacqueline Morgan [b,*]

[a] *Department of Mathematics and Applications R. Caccioppoli, University of Naples Federico II, Via Claudio, 80125 Naples, Italy*
[b] *Department of Economics and Statistics & CSEF, University of Naples Federico II, Via Cinthia, 80126 Naples, Italy*

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ABSTRACT

The great interest into hierarchical optimization problems and the increasing use of game theory in many economic or engineering applications led to investigate scalar bilevel problems in which the upper level is an optimization problem and the lower level is a parametrized quasi-variational inequality. In this paper, we analyze the convergence of the sequences of infima and minima for the upper level when the data of the problem are perturbed. First, we show that general results on the convergence of the infima and minima may not be possible. Thus, we introduce suitable concepts of regularized semi-quasivariational optimistic bilevel problems and we study, in Banach spaces, the convergence properties of the infima and minima to these regularized problems in the presence or not of perturbations.

1. Introduction

Given a Hausdorff topological space (X, τ), a nonempty τ-closed set $H \subseteq X$ and a real Banach space E with dual E^*, we consider, for any $x \in H$, the parametric quasi-variational inequality (q.v.i. for brevity)

$$Q(x) \quad \text{find } u \in S(x,u) \quad \text{such that} \quad \big\langle A(x,u), u - w\big\rangle \leq 0 \quad \forall w \in S(x,u) \tag{1}$$

where $K \subseteq E$ is nonempty, closed and convex, A is an operator from $H \times K$ to E^* and S is a set-valued (or multi-valued) map from $H \times K$ to K such that $S(x,u) \neq \emptyset$ for each $x \in H$ and $u \in K$.

We denote by $\mathcal{Q}(x)$ the set of solutions to the problem $Q(x)$ and we remark that the solution map $\mathcal{Q} : x \in H \to \mathcal{Q}(x)$ is set-valued even under restrictive assumptions [4].

* Corresponding author.
E-mail addresses: lignola@unina.it (M.B. Lignola), morgan@unina.it (J. Morgan).

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The problem $Q(x)$ is a particular case of a more general parametric problem considered in [17] and [19]

$$\text{find } u \in K \quad \text{such that} \quad h(x,u,w) + \phi(x,u,u) \leq \phi(x,u,w) \quad \forall w \in K \tag{2}$$

where $h : H \times K \times K \to \mathbb{R}$ and $\phi : H \times K \times K \to \mathbb{R} \cup \{+\infty\}$ are single-valued maps.

Indeed, it suffices to consider the function ϕ defined by the indicator function ψ of the set $S(x,u)$ which takes the value 0 on $S(x,u)$ and the value $+\infty$ otherwise, that is $\phi(x,u,w) = \psi_{S(x,u)}(w)$, and the function h defined by $h(x,u,w) = \langle A(x,u), u - w\rangle$. Also observe that when $h(x,u,w) = \langle A(x,u), u - w\rangle$ and $\phi(x,u,w) = \psi_{T(x)}(w)$, where T is a set-valued map from H to K, problem (2) becomes a parametric variational inequality [15] (v.i. for brevity).

The great interest into hierarchical optimization problems and the increasing use of game theory in many economic or engineering applications ([25,11,7,10,26], etc.) led to investigate bilevel problems in which the upper level is an optimization problem and the lower level is a parametrized quasi-variational inequality. Such a problem, that we denominate *Semi-quasivariational Optimistic Bilevel Problem*, consists in finding $(x_o, u_o) \in H \times K$ such that

$$(SB) \quad u_o \in \mathcal{Q}(x_o) \quad \text{and} \quad f(x_o,u_o) = \min_{x\in H} \min_{u\in\mathcal{Q}(x)} f(x,u)$$

where f is a function from $H \times K$ to $\mathbb{R} \cup \{+\infty\}$.

Here, in analogy with the term *semivectorial* introduced in [5], the term "semi-quasivariational" means that at the lower level a parametric q.v.i. is solved (by one or more followers) meanwhile at the upper level the leader solves a scalar optimization problem with constraints determined by the set of solutions to the lower level problem. Moreover, the term "optimistic", which is in line with [25] and [9], could be replaced by the term "strong" as in [6] and [24].

The set of solutions and the infimum of the problem (SB) are denoted by $\mathcal{M}$ and φ respectively, so we have

$$(x_o,u_o) \in \mathcal{M} \quad \Longleftrightarrow \quad u_o \in \mathcal{Q}(x_o) \quad \text{and} \quad f(x_o,u_o) = \min_{x\in H} \min_{u\in\mathcal{Q}(x)} f(x,u) \tag{3}$$

and

$$\varphi = \inf_{x\in H} \inf_{u\in\mathcal{Q}(x)} f(x,u).$$

In this paper, motivated as in [21, Introduction], given a sequence of operators $(A_n)_n$, a sequence of functions $(f_n)_n$ and a sequence of set-valued maps $(S_n)_n$, we investigate the convergence of the infima φ_n and of the minima $\mathcal{M}_n$ for the problems $(SB)_n$ where

$$(x_n,u_n) \in \mathcal{M}_n \quad \Longleftrightarrow \quad u_n \in \mathcal{Q}_n(x_n) \quad \text{and} \quad f_n(x_n,u_n) = \min_{x\in H} \min_{u\in\mathcal{Q}_n(x)} f_n(x,u)$$

and

$$\varphi_n = \inf_{x\in H} \inf_{u\in\mathcal{Q}_n(x)} f(x,u).$$

The convergence of the infima φ_n has been investigated in [21] when $V = \mathbb{R}^k$ and the q.v.i. $Q(x)$ is nothing else but a variational inequality. The pessimistic case where the leader solves a minsup problem has been studied in [22].

Here, in the setting of infinite dimensional spaces, we have two aims: one is to investigate the convergence of the infima of semi-quasivariational optimistic bilevel problems, one is to study the convergence of their solutions.