J. Math. Anal. Appl. 424 (2015) 423–436

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Decay results for viscoelastic equations with coupled nonlinear boundary conditions

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ARTICLE INFO

Article history:
Received 10 August 2014
Available online 13 November 2014
Submitted by P. Yao

Keywords:
Viscoelastic equations
Coupled boundary conditions
Decay result

ABSTRACT

In this paper, we study the initial–boundary value problem of viscoelastic equations with coupled nonlinear boundary conditions. By using the energy perturbation method, we establish decay results of the solution as time goes to infinity.

1. Introduction

Many phenomena in physics and engineering give rise to problems that deal with evolution equations, which are modeled by partial differential equations. The global existence, asymptotic behavior, blow-up for the solutions of such problems have been considerably stimulated in recent years. In this paper, we are concerned with the following initial–boundary value problem,

$$\begin{cases} |u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \displaystyle\int_0^t g_1(t-\tau)\Delta u(x,\tau)d\tau = 0, & (x,t) \in \Omega \times (0,\infty), \\ |v_t|^\rho v_{tt} - \Delta v - \Delta v_{tt} + \displaystyle\int_0^t g_2(t-\tau)\Delta v(x,\tau)d\tau = 0, & (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = v(x,t) = 0, & (x,t) \in \Gamma_0 \times (0,\infty), \\ \dfrac{\partial u_{tt}}{\partial \nu} + \dfrac{\partial u}{\partial \nu} - \displaystyle\int_0^t g_1(t-\tau)\frac{\partial u(\tau)}{\partial \nu}d\tau + f(u,v) = 0, & (x,t) \in \Gamma_1 \times (0,\infty), \\ \dfrac{\partial v_{tt}}{\partial \tau} + \dfrac{\partial v}{\partial \tau} - \displaystyle\int_0^t g_2(t-\tau)\frac{\partial v(\tau)}{\partial \tau}d\tau + k(u,v) = 0, & (x,t) \in \Gamma_1 \times (0,\infty), \\ u(0,x) = u_0(x), \qquad u_t(0,x) = u_1(x), & x \in \Omega, \\ v(0,x) = v_0(x), \qquad v_t(0,x) = v_1(x), & x \in \Omega, \end{cases} \tag{1.1}$$

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http://dx.doi.org/10.1016/j.jmaa.2014.11.025
0022-247X/

where Ω is a bounded domain of R^n $(n \geq 1)$ with smooth boundary $\partial\Omega$, $\Gamma_0 \cup \Gamma_1 = \partial\Omega$, $\overline{\Gamma_0} \cap \overline{\Gamma_1} = \emptyset$, $meas(\Gamma_0) > 0$, $0 < \rho \leq \frac{2}{n-2}$ for $n \geq 3$, $\rho > 0$ for $n = 1, 2$. The functions g_1, g_2, f and k are nonlinear functions.

The pioneer work of Dafermos [5] in 1970 discussed a one-dimensional viscoelastic problem, established some existence and stability results with some smooth monotone decreasing relaxation functions. Hrusa [8] established several global existence and asymptotic stability results for a semilinear hyperbolic Volterra equation which allows the initial data to be large. In 1994, Rivera [14] gave very important contribution, considered equations for linear isotropic viscoelastic solids of integral type, and established exponential decay and polynomial decay in a bounded domain and in the whole space respectively. After that, many results of existence and long-term behavior have been established. Messaoudi [11] considered a nonlinear viscoelastic wave equation

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + u_t|u_t|^{m-2} = u|u|^{p-2}, \tag{1.2}$$

with source term, damping term and with Dirichlet boundary condition. He got blow-up result for solutions with negative initial energy and $m < p$. He also gave a global existence result for arbitrary initial (in the appropriate space) if $m \geq p$.

Cavalcanti and Oquendo [1] discussed

$$u_{tt} - k_0\Delta u + \int_0^t div\big[a(x)g(t-\tau)\nabla u(\tau)\big]d\tau + b(x)h(u_t) + f(u) = 0, \tag{1.3}$$

under some conditions on the relaxation function g and $a(x) + b(x) \geq \delta > 0$. They showed an exponential stability result when the relaxation function g is decaying exponentially and the function h is linear, and showed a polynomial stability when g is decaying polynomially and h is non-linear.

Rivera and Naso [15] considered a viscoelastic systems with nondissipative kernels, and showed that if the kernel function decays exponentially to zero, then the solution decays exponentially to zero. On the other hand, if the kernel function decays polynomially as t^{-p}, then the corresponding solution also decays polynomially to zero with the same rate of decay.

Cavalcanti et al. [2] discussed a quasilinear initial–boundary value problem of equation

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-s)\Delta u(\tau)d\tau - \gamma\Delta u_t = bu|u|^{p-2}, \tag{1.4}$$

with Dirichlet boundary condition, where $\rho > 0$, $\gamma \geq 0$, $p \geq 2$, $b = 0$. An exponential decay result for $\gamma > 0$ and $b = 0$ has been obtained. For $\gamma = 0$ and $b > 0$, Messaoudi and Tatar [12] showed that there exists an appropriate set, called stable set, such that if the initial data are in stable set, the solution continues to live there forever, and the solution goes to zero with an exponential or polynomial rate depending on the decay rate of relaxation function.

Guo et al. [6] studied a viscoelastic wave equation with supercritical source and damping terms

$$u_{tt} - k(0)\Delta u - \int_0^\infty k'(\tau)\Delta u(t-\tau)d\tau + g(u_t) = f(u). \tag{1.5}$$