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Cauchy problem for an isentropic magnetogasdynamic system

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ABSTRACT

This paper investigates the Cauchy problem for an isentropic magnetogasdynamic system. Under certain reasonable hypotheses on the initial data, we obtain the global existence and uniqueness of the C^1 solution to the system. Meanwhile, when the hypotheses on the initial data do not hold, we obtain the blow-up phenomena of the C^1 solution to the system. The bounds of the solution are shown to depend on the parameter ν, which characterizes a one-dimensional plane flow ($\nu = 0$) or a three-dimensional cylindrically symmetric flow ($\nu = 1$); it is shown that the existence of the finite time singularity is significantly influenced by the magnetic field strength present in the flow along with the initial data.

1. Introduction

In this paper, we are concerned with the global existence of smooth solution to the Cauchy problem for magnetogasdynamics (see [2,9])

$$\begin{cases} \rho_t + (\rho u)_x + \dfrac{\nu u \rho}{x} = 0, \\ u_t + u u_x + \dfrac{1}{\rho}\left(p + B^2/2\mu\right)_x = 0, \quad t > 0,\ x \in \mathbb{R}, \\ \rho(0,x) = \rho_0(x), \qquad u(0,x) = u_0(x), \quad x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where $\rho > 0$, u, $p \geq 0$, and B denote, respectively, the density, particle velocity, pressure and the magnetic field with μ as the magnetic permeability being treated as a constant; the variable t stands for time and the variable x denotes the spatial coordinate, being either axial in flows with planar ($\nu = 0$) geometry or radial in cylindrically symmetric ($\nu = 1$) configuration. The magnetic field is transverse to the flow direction; indeed, in a cylindrically symmetric flow it is along the axis of symmetry. In (1.1), p and B are known

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functions defined as $p = k_1\rho^\gamma$, $B = k_2\rho$ with k_1, k_2 as positive constants; γ is the adiabatic constant that lies in the range $1 < \gamma \le 2$ for most gases. Here ν may take values 0 or 1. Recently, the Riemann problem and elementary waves interactions for system (1.1) with a planar case ($\nu = 0$) were studied by Sekhar et al. (see [8]). The discussion of global solution to the Cauchy problem (1.1) for the one dimensional planar flow of an isotropic fluid is largely complete [4,6]; for the related work on such a system, we refer to [1,5,10,12,13] and the references therein.

The purpose of this paper is to study the Cauchy problem for system (1.1) with smooth C^1 bounded initial data. Indeed, we study the global existence and uniqueness of the C^1 solution to Cauchy problem under certain reasonable hypotheses on the initial data; when the hypotheses on the initial data do not hold, we discuss the blow-up phenomenon of the C^1 solution to the system (1.1). By using the Riemann invariants, system (1.1) can be rewritten as a diagonal form. For this purpose, we first set $U = (\rho \quad u)^T$, then for smooth solutions, system (1.1) is equivalent to

$$\begin{pmatrix} \rho \\ u \end{pmatrix}_t + \begin{pmatrix} u & \rho \\ \frac{w^2}{\rho} & u \end{pmatrix} \begin{pmatrix} \rho \\ u \end{pmatrix}_x + \begin{pmatrix} \frac{\nu u \rho}{x} \\ 0 \end{pmatrix} = 0, \tag{1.2}$$

where $w = (c^2 + b^2)^{1/2}$ is the magneto-acoustic speed with $c = (p'(\rho))^{\frac{1}{2}}$ as the local sound speed and $b = (B^2(\rho)/\mu\rho)^{1/2}$ the Alfven speed. Here, prime denotes differentiation with respect to ρ. Remember that $p = k_1\rho^\gamma$ with $1 < \gamma \le 2$ and $B = k_2\rho$, a short calculation shows that

$$2w'(\rho)\rho \le w. \tag{1.3}$$

Next, it is easy to see that the eigenvalues of A are $\lambda_1 = u - w$ and $\lambda_2 = u + w$. Thus, the system (1.2) is strictly hyperbolic when $w > 0$. Now, we introduce the Riemann invariants for the system (1.2) corresponding to the eigenvalues λ_1 and λ_2, respectively

$$R^- = u - \int_{\rho_*}^{\rho} \frac{w(y)}{y} dy, \qquad R^+ = u + \int_{\rho_*}^{\rho} \frac{w(y)}{y} dy, \tag{1.4}$$

where $\rho_* > 0$ is a fixed number. By (1.4), it is easy to check that

$$\frac{\partial \lambda_1}{\partial R^-} = \frac{\partial \lambda_2}{\partial R^+} = \frac{1}{2} + \frac{w'(\rho)\rho}{2w(\rho)}, \qquad \frac{\partial \lambda_1}{\partial R^+} = \frac{\partial \lambda_2}{\partial R^-} = \frac{1}{2} - \frac{w'(\rho)\rho}{2w(\rho)}. \tag{1.5}$$

Meanwhile, for smooth solutions, system (1.2) is equivalent to

$$\begin{cases} R^-_t + \lambda_1 R^-_x = \dfrac{\nu u w}{x}, \\ R^+_t + \lambda_2 R^+_x = -\dfrac{\nu u w}{x}, \quad t > 0,\ x \in \mathbb{R}, \end{cases} \tag{1.6}$$

subject to bounded and differentiable initial data

$$R^-(0, x) = u_0 - \int_{\rho_*}^{\rho_0} \frac{w(y)}{y} dy, \qquad R^+(0, x) = u_0 + \int_{\rho_*}^{\rho_0} \frac{w(y)}{y} dy. \tag{1.7}$$

Through this reformulated system, there exists a uniform invariant region for the system (1.1) (see [7]). Thus there exist constants $0 < \rho_{\min} < \rho_{\max}$ and $u_{\min} < u_{\max}$, depending only on the initial data (ρ_0, u_0), such that