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## Cauchy problem for an isentropic magnetogasdynamic system

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#### ABSTRACT

This paper investigates the Cauchy problem for an isentropic magnetogasdynamic system. Under certain reasonable hypotheses on the initial data, we obtain the global existence and uniqueness of the  $C^1$  solution to the system. Meanwhile, when the hypotheses on the initial data do not hold, we obtain the blow-up phenomena of the  $C^1$  solution to the system. The bounds of the solution are shown to depend on the parameter  $\nu$ , which characterizes a one-dimensional plane flow ( $\nu = 0$ ) or a three-dimensional cylindrically symmetric flow ( $\nu = 1$ ); it is shown that the existence of the finite time singularity is significantly influenced by the magnetic field strength present in the flow along with the initial data.

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### 1. Introduction

In this paper, we are concerned with the global existence of smooth solution to the Cauchy problem for magnetogasdynamics (see [2,9])

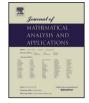
$$\begin{cases} \rho_t + (\rho u)_x + \frac{\nu u \rho}{x} = 0, \\ u_t + u u_x + \frac{1}{\rho} (p + B^2 / 2\mu)_x = 0, \quad t > 0, \ x \in \mathbb{R}, \\ \rho(0, x) = \rho_0(x), \qquad u(0, x) = u_0(x), \quad x \in \mathbb{R}, \end{cases}$$
(1.1)

where  $\rho > 0$ ,  $u, p \ge 0$ , and B denote, respectively, the density, particle velocity, pressure and the magnetic field with  $\mu$  as the magnetic permeability being treated as a constant; the variable t stands for time and the variable x denotes the spatial coordinate, being either axial in flows with planar ( $\nu = 0$ ) geometry or radial in cylindrically symmetric ( $\nu = 1$ ) configuration. The magnetic field is transverse to the flow direction; indeed, in a cylindrically symmetric flow it is along the axis of symmetry. In (1.1), p and B are known

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functions defined as  $p = k_1 \rho^{\gamma}$ ,  $B = k_2 \rho$  with  $k_1$ ,  $k_2$  as positive constants;  $\gamma$  is the adiabatic constant that lies in the range  $1 < \gamma \leq 2$  for most gases. Here  $\nu$  may take values 0 or 1. Recently, the Riemann problem and elementary waves interactions for system (1.1) with a planar case ( $\nu = 0$ ) were studied by Sekhar et al. (see [8]). The discussion of global solution to the Cauchy problem (1.1) for the one dimensional planar flow of an isotropic fluid is largely complete [4,6]; for the related work on such a system, we refer to [1,5,10,12,13] and the references therein.

The purpose of this paper is to study the Cauchy problem for system (1.1) with smooth  $C^1$  bounded initial data. Indeed, we study the global existence and uniqueness of the  $C^1$  solution to Cauchy problem under certain reasonable hypotheses on the initial data; when the hypotheses on the initial data do not hold, we discuss the blow-up phenomenon of the  $C^1$  solution to the system (1.1). By using the Riemann invariants, system (1.1) can be rewritten as a diagonal form. For this purpose, we first set  $U = (\rho \ u)^T$ , then for smooth solutions, system (1.1) is equivalent to

$$\begin{pmatrix} \rho \\ u \end{pmatrix}_{t} + \begin{pmatrix} u & \rho \\ \frac{w^{2}}{\rho} & u \end{pmatrix} \begin{pmatrix} \rho \\ u \end{pmatrix}_{x} + \begin{pmatrix} \frac{\nu u \rho}{x} \\ 0 \end{pmatrix} = 0,$$
(1.2)

where  $w = (c^2 + b^2)^{1/2}$  is the magneto-acoustic speed with  $c = (p'(\rho))^{\frac{1}{2}}$  as the local sound speed and  $b = (B^2(\rho)/\mu\rho)^{1/2}$  the Alfven speed. Here, prime denotes differentiation with respect to  $\rho$ . Remember that  $p = k_1 \rho^{\gamma}$  with  $1 < \gamma \leq 2$  and  $B = k_2 \rho$ , a short calculation shows that

$$2w'(\rho)\rho \le w. \tag{1.3}$$

Next, it is easy to see that the eigenvalues of A are  $\lambda_1 = u - w$  and  $\lambda_2 = u + w$ . Thus, the system (1.2) is strictly hyperbolic when w > 0. Now, we introduce the Riemann invariants for the system (1.2) corresponding to the eigenvalues  $\lambda_1$  and  $\lambda_2$ , respectively

$$R^{-} = u - \int_{\rho_{*}}^{\rho} \frac{w(y)}{y} dy, \qquad R^{+} = u + \int_{\rho_{*}}^{\rho} \frac{w(y)}{y} dy, \qquad (1.4)$$

where  $\rho_* > 0$  is a fixed number. By (1.4), it is easy to check that

$$\frac{\partial\lambda_1}{\partial R^-} = \frac{\partial\lambda_2}{\partial R^+} = \frac{1}{2} + \frac{w'(\rho)\rho}{2w(\rho)}, \qquad \frac{\partial\lambda_1}{\partial R^+} = \frac{\partial\lambda_2}{\partial R^-} = \frac{1}{2} - \frac{w'(\rho)\rho}{2w(\rho)}.$$
(1.5)

Meanwhile, for smooth solutions, system (1.2) is equivalent to

$$\begin{cases} R_t^- + \lambda_1 R_x^- = \frac{\nu u w}{x}, \\ R_t^+ + \lambda_2 R_x^+ = -\frac{\nu u w}{x}, \quad t > 0, \ x \in \mathbb{R}, \end{cases}$$
(1.6)

subject to bounded and differentiable initial data

$$R^{-}(0,x) = u_0 - \int_{\rho_*}^{\rho_0} \frac{w(y)}{y} dy, \qquad R^{+}(0,x) = u_0 + \int_{\rho_*}^{\rho_0} \frac{w(y)}{y} dy.$$
(1.7)

Through this reformulated system, there exists a uniform invariant region for the system (1.1) (see [7]). Thus there exist constants  $0 < \rho_{\min} < \rho_{\max}$  and  $u_{\min} < u_{\max}$ , depending only on the initial data  $(\rho_0, u_0)$ , such that

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