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On the number of limit cycles in discontinuous piecewise linear differential systems with two pieces separated by a straight line

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ABSTRACT

In this paper we study the maximum number N of limit cycles that can exhibit a planar piecewise linear differential system formed by two pieces separated by a straight line. More precisely, we prove that this maximum number satisfies $2 \leq N \leq 4$ if one of the two linear differential systems has its equilibrium point on the straight line of discontinuity.

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1. Introduction and statement of the main result

The study of piecewise linear differential systems goes back to Andronov, Vitt and Khaikin [1] and still continues to receive attention by researchers. These last years a renewed interest has appeared in the mathematical community working in differential equations for understanding the dynamical richness of the piecewise linear differential systems, because these systems are widely used to model processes appearing in electronics, mechanics, economy, etc., see for instance the books of di Bernardo, Budd, Champneys and Kowalczyk [3], and Simpson [25], and the survey of Makarenkov and Lamb [23], and the hundreds of references quoted in these last three works.

We recall that a *limit cycle* is a periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system.

The simplest possible continuous but nonsmooth piecewise linear differential systems are the ones having only two pieces separated by a straight line. In 1990 Lum and Chua [22] conjectured that a continuous piecewise linear vector field in the plane with two pieces has at most one limit cycle. We note that even in this apparent simple case, only after a difficult analysis it was possible to prove the existence of at most

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one limit cycle, thus in 1998 this conjecture was proved by Freire, Ponce, Rodrigo and Torres [7]. There are two reasons that make difficult the analysis of these differential systems. First, even one can easily integrate the solutions of every linear differential system, the time that an orbit expends in each half-plane governed by each linear differential system is in general unknown, consequently the matching of the corresponding solutions is a difficult problem. Second, the number of parameters to consider in order to be sure that we take into account all possible cases is in general not small. Of course, these difficulties increase when we work with discontinuous piecewise linear differential systems. Recently, a new an easier proof that at most one limit cycle exists for the continuous piecewise linear differential systems with two pieces separated by a straight line has been done by Llibre, Ordóñez and Ponce in [19].

The objective of this paper is to study the problem of Lum and Chua but now for the class of discontinuous piecewise linear differential systems in the plane with two pieces separated by a straight line. In some sense this problem can be seen as an extension of the 16th Hilbert's problem to the discontinuous piecewise linear differential systems in the plane with two pieces separated by a straight line. We recall that the 16th Hilbert's problem essentially asks for the maximum number of limit cycles that a polynomial differential system in the plane can have in function of the degree of the system. For the moment this problem remains open, for more details on the 16th Hilbert's problem see for instance [12,16,18].

Several authors tried to determine the maximum number of nested limit cycles surrounding a unique equilibrium point for the class of all discontinuous piecewise linear differential systems with two pieces separated by a straight line. Thus in the paper of Han and Zhang [11] some results about the existence of two limit cycles appeared, so that the authors conjectured that the maximum number of limit cycles for this class of piecewise linear differential systems is exactly two. But Huan and Yang in [13] provided numerical evidence about the existence of three nested limit cycles surrounding a unique equilibrium. Llibre and Ponce in [20] inspired in the numerical example of [13] proved that there are discontinuous piecewise linear differential systems with two pieces separated by a straight line having three limit cycles. Later on other authors obtained also three limit cycles for those differential systems following different ways, see the papers of Braga and Mello [4], of Buzzi, Pessoa and Torregrosa [5], and of Freire, Ponce and Torres [9].

The linear differential systems that we consider in every half-plane extended to the full plane is either a focus (F) (we include in this class of foci the centers), or a node (N), or a saddle (S). We recall that there are three classes of linear nodes: nodes with different eigenvalues, nodes with equal eigenvalues whose linear part does not diagonalize, and nodes with equal eigenvalues whose linear part diagonalize, called *star nodes*. Clearly if a piecewise linear differential system with two pieces separated by a straight line has a star node, this prevents the existence of periodic orbits.

An equilibrium point p of a linear differential system defined in a half-plane having in the full plane a node, a focus or a saddle is *real* when p belongs to the closure of the half-plane where the system is defined the mentioned linear differential system, and p is called *virtual* otherwise.

We distinguish six classes or types of planar discontinuous piecewise linear differential systems: FF, FN, FS, NN, NS and SS. Inside these classes and *in this paper we only consider limit cycles surrounding a unique equilibrium point or a unique sliding segment.* So we do not consider sliding limit cycles. Now we recall the definitions of sliding segment and non-sliding limit cycle, for more details on these definitions see for instance [10] and [26].

Let $Z = (X, Y)$ be a discontinuous piecewise linear differential vector field with two pieces separated by a straight line Σ, in one piece we have the linear vector field X and in the other the linear vector field Y. Following Filippov [6] we distinguish three open regions in the discontinuity straight line Σ.

1) The *sliding region* Σ^{sl} where the vectors $X(p)$ and $Y(p)$ with $p \in \Sigma$ point inward Σ.
2) The *escaping region* Σ^{e} where the vectors $X(p)$ and $Y(p)$ with $p \in \Sigma$ point outward Σ.
3) The *sewing region* Σ^{s} where the vectors $X(p)$ and $Y(p)$ with $p \in \Sigma$ point to the same direction and are transverse to Σ.

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