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Traveling waves and competitive exclusion in models of resource competition and mating interference

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ABSTRACT

In this article, we study reaction–diffusion models for two closed related biological species (u and v) under resource competition and mating interference from v-species to u-species. The effect of one-sided sexual competition makes the trivial state and u-dominance state both unstable, while the v-dominance state is shown to be asymptotically stable with attraction regions and convergent rates depending on the biological parameters. Using the upper–lower solution method, we further prove that for a family of wave speeds, there exist traveling wave solutions connecting the u-dominance state and the v-dominance state at $\xi \to \mp\infty$. This confirms an earlier conjecture that unbalanced mating interference will lead to competitive exclusion. These results can also be obtained on an extended model with instantaneous effects of resource competition and temporal delay on mating interference. Through a transformation into three-equation system, we prove that the temporal delay does not affect the stability of the steady states and the existence of the traveling waves, but causes changes on the attraction regions and convergence rates. Finally, numerical simulations are also presented to illustrate the theoretical results.

1. Introduction

Reaction–diffusion models with Lotka–Volterra equations have been widely used for interspecies resource competition, and are extensively studied for coexistence and competitive exclusion, asymptotic stability, and traveling wave solutions [1,4,8,10–13,16,17,28]. Biological research [3,5,7,14,21,23–25,27] has shown that among closely related species, many competitive interactions occur that are likely to involve both resource competition and mating interference. In these cases, sexual competition (male mating competition and/or female mating preference) enters population dynamics as a negative result of interspecific mating behavior [7,27]. As for many insects, interspecific mating has a negligible effect on males, but results in reproductive

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failure for females [3,5,14,27]. Therefore, sexual and resource competition are known to affect birth rates independently, and resource competition alone affects mortality rates.

In this paper, we study a competition model of two closely related species in infinite spatial domain $\mathbb{R}$ where the movability and sex ratios are the same. Both species compete for resource, and the males of one species (v) have mating interference on the females of another species (u). The model was originally proposed as an ordinary differential equation system by Yoshimura and Clark [27] with possibility of interspecies mating on both species, with the prediction that strong and unbalanced interspecies mating simply leads to competitive exclusion. The persistence and competitive exclusion of this model as a reaction–diffusion system in a bounded habitat was also studied in [9]. After scaling of population densities and interaction parameters (see details in [9]), the following reaction–diffusion system is considered in this article:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = u\big[-\alpha - a_1 u - a_2 v + (\rho - b_1 u - b_2 v)\dfrac{u}{u+\beta v}\big], & (x,t) \in \mathbb{R} \times \mathbb{R}^+ \\ \dfrac{\partial v}{\partial t} - \dfrac{\partial^2 v}{\partial x^2} = v[1 - ku - v]. \end{cases} \tag{1.1}$$

In model (1.1), the quantities $u(x,t)$ and $v(x,t)$ are scaled population densities of females in the two species at $t > 0$ and $x \in \mathbb{R}$, with mating interference from males in species v on females in species u. In the first equation, $\rho > 0$ and $\alpha > 0$ are the intrinsic birth rate and death rate of the u species. a_i and b_i are the resource-competition coefficients affecting the birth and death rates for u species. $\frac{u}{u+\beta v}$ denotes the reduction factor on birth rate of u species caused by sexual competition. The sexual-competition coefficient $0 < \beta \le 1$ is the comparative rate of the interspecies mating. $\beta = 0$ gives the case of complete sexual isolation (Lotka–Volterra model for resource competition only), and $\beta = 1$ indicates that the u females cannot discriminate against v males. The second equation in the system is a typical scaled Lotka–Volterra equation for v species which only experiences resource competition from u species. As seen in [9], the population density functions can be scaled such that $a_1 + b_1 = 1$. We also assume that after scaling, $0 < a_2 + b_2 < \min\{1, r\}$ and $0 < k < 1$. This implies that the intra-species resource-competition rates are larger than inter-species ones such that in the absence of the sexual competition ($\beta = 0$), the Lotka–Volterra model has a stable coexistence state.

While the intrinsic growth rate for v species is scaled as 1, we assume that the u-species has its intrinsic growth rate r with $0 < r = \rho - \alpha < 1/k$. The following assumptions are made throughout this paper:

$$\textbf{H1}: \quad a_1 + b_1 = 1, \quad 0 < a_2 + b_2 < \min\{1, r\}, \quad 0 < \beta \le 1,$$
$$0 < k < 1, \quad 0 < r = \rho - \alpha < 1/k.$$

In Section 2, we will give a thorough analysis on the asymptotic stability of the equilibrium states in model (1.1). Through suitably constructed upper and lower solutions, we prove that the extinction state $(0,0)$ and u-dominance state $(r,0)$ are unstable all the time, while the v-dominance state $(0,1)$ is asymptotically stable with specifically given attraction region and exponential rates. We also show that the biological parameters (α, β, ρ, k and a_1) affect the attraction area of the v-dominance state $(0,1)$. For all initial density functions falling into the attraction area given by Theorem 2.3, the solution of (1.1) converges to $(0,1)$ uniformly on $\mathbb{R}$ with the rate e^{-pt} (p depends on α and k) as $t \to \infty$.

Section 3 is devoted to the existence and asymptotic rates of traveling wave solutions in the competition system (1.1) with mating interference. We will show that under an assumption (**H3**) on the ecological parameters, the dominance shifting from u-species to v-species is in the traveling wave format and evolves with both time and space. Using the method of upper–lower solutions and monotone iteration, we prove (in Theorem 2.3) that for each wave speed $c \ge 2\sqrt{1 - kr}$, there exists a positive and monotone traveling wave solution $(u(x+ct), v(x+ct))$ of (1.1) with $(u(-\infty), v(-\infty)) = (r, 0)$ and $(u(\infty), v(\infty)) = (0, 1)$, with explicit