J. Math. Anal. Appl. 424 (2015) 774–784

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Perturbations of symmetric elliptic Hamiltonians of degree four in a complex domain

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A R T I C L E   I N F O

Article history:
Received 21 January 2014
Available online 11 November 2014
Submitted by C.E. Wayne

Keywords:
Limit cycles
16th Hilbert problem
Zeros of elliptic integrals depending on parameters

A B S T R A C T

The cyclicity of the exterior period annulus of the asymmetrically perturbed Duffing oscillator is a well known problem extensively studied in the literature. In the present paper we provide a complete bifurcation diagram for the number of the zeros of the associated Melnikov function in a suitable complex domain.

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1. Introduction

Consider the asymmetrically perturbed Duffing oscillator

$$X_{\lambda,\nu} : \begin{cases} \dot{x} = y \\ \dot{y} = x - x^3 + \nu x^2 + \lambda_0 y + \lambda_1 xy + \lambda_2 x^2 y \end{cases} \tag{1}$$

in which ν, λ_i are small real parameters. For $\nu = \lambda_0 = \lambda_1 = \lambda_2 = 0$ the system is integrable, with a first integral

$$H = \frac{y^2}{2} - \frac{x^2}{2} + \frac{x^4}{4}$$

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http://dx.doi.org/10.1016/j.jmaa.2014.11.018
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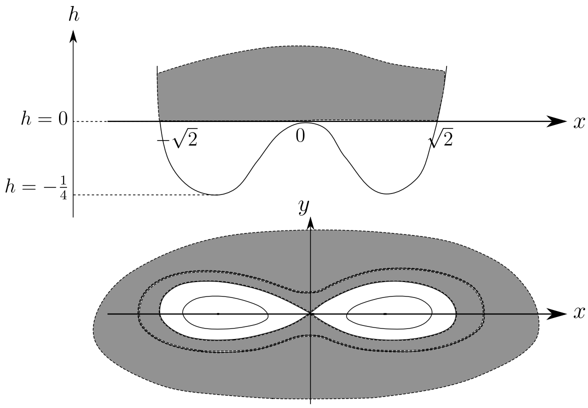

Fig. 1. Phase portrait of X_0 on the (x, y)-plane and the graph of $h = -\frac{x^2}{2} + \frac{x^4}{4}$.

and its phase portrait is shown in Fig. 1. Alternatively, the system (1) defines a real plane foliation by the formula

$$d\left(H - \nu\frac{x^3}{3}\right) + \left(\lambda_0 + \lambda_1 x + \lambda_2 x^2\right) y dx = 0. \tag{2}$$

The maximal number of limit cycles, which bifurcate from the exterior period annulus of X_0 with respect to the perturbation $X_{\lambda,\nu}$ is equal to two, as it has been shown by Iliev and Perko [7] and Li, Mardesic and Roussarie [9]:

Theorem 1. *(See [7,9].) The cyclicity of the exterior period annulus $\{(x, y) \in \mathbb{R}^2 : H(x, y) > 0\}$ of $dH = 0$ with respect to the perturbation (1) equals two.*

Remark 1. The above theorem claims that from any compact, contained in the open exterior period annulus $\{(x, y) \in \mathbb{R}^2 : H(x, y) > 0\}$, bifurcate at most two limit cycles. It says nothing about the limit cycles bifurcating from the separatrix eight-loop or from infinity (i.e. the equator of the Poincaré sphere).

Let $\{\gamma(h)\}_h$ be the continuous family of exterior ovals of the non-perturbed system, where

$$\gamma(h) \subset \{H = h\}$$

and consider the complete elliptic integrals

$$I_i = \oint_{\gamma(h)} x^i y dx. \tag{3}$$

It has been shown in [7], that if we restrict our attention to a one parameter deformation

$$\lambda_i = \lambda_i(\varepsilon), \qquad \nu = \nu(\varepsilon)$$

then the first non-vanishing Poincaré–Pontryagin–Melnikov function M_k (governing the bifurcation of limit cycles) is given by a linear combination of the complete elliptic integrals of first and second kind I_0, I_2, I_4'

$$M_k(h) = \lambda_{0k} I_0(h) + \lambda_{2k} I_2(h) + \lambda_{4k} I_4'(h). \tag{4}$$

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