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Pricing American continuous-installment options under stochastic volatility model

Guohe Deng

School of Mathematics and Statistics, Guangxi Normal University, Guilin 541004, China

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ABSTRACT

This paper presents an integral equation approach for pricing American continuous-installment options when the stock price follows Heston's stochastic volatility model. By exploiting a log-linear relationship of the free boundary function with respect to volatility changes and using the decomposition technique and Fourier inversion transform, we drive integral expressions of the initial premium along with the optimal stopping and early exercise boundaries for this option. This offers a system of nonlinear Volterra integral equations for determining the two free boundaries, which can be used to estimate the option price. Numerical integration technique accompanied with the Newton–Rahpson iteration procedure is proposed for solving the integral equations. The method is implemented, and some numerical examples are provided to examine the boundary properties and the option price behavior. The computational efficiency of this method is also considered.

1. Introduction

In contrast to a traditional option contract, installment option (IO) requires the option holder paying a smaller amount of up-front premium at the time of purchase and then offers a sequence of "installments" paid up to a fixed maturity date. To keep the option alive, the holder must continue to pay the premiums, although he/she has the right to terminate the option contract by halting the payments at any time before the maturity date. This reduces considerably the cost of entering into a hedging strategy and the liquidity risk associated with other OTC derivatives. There are two basic types: one is a discrete-installment option in which the premium payments are paid at pre-specified dates, the other is a continuous-installment option in which a constant stream of installments is paid at a certain rate per unit time. For an American continuous-installment option, the holder can choose at any time to stop making installment payments by either exercising the option or stopping the option contract during the life time of the option, which will results in two free boundaries named the optimal stopping and early exercise boundaries.

E-mail address: dengguohe@mailbox.gxnu.edu.cn.

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There is not much literature on the pricing of the installment options. Under the celebrated Black and Scholes [6] model (BS model), no-arbitrage bounds and static hedging strategies for the European-style discrete-installment options were derived by Davis et al. [14,15], then by Ben-Ameur et al. [5] for value the American discrete-installment option. Griebsch et al. [19] derived a closed-form solution to the value of the discrete-installment option and examined the limiting case with continuous payment plan.

For the continuous-installment options, Alobaidi et al. [4] used a partial Laplace transform to the inhomogeneous partial differential equation (PDE) for the option value function and analyzed asymptotic properties of the optimal stopping boundary close to maturity. Ciurlia and Roko [13] derived an integral representation of the initial premium for the American-style and applied the multipiece exponential function (MEF) method studied by Ju [22] to the valuation formulas. Alobaidi and Mallier [3] considered the behavior of the continuous-installment options close to expiry by applying an asymptotic expansion. More recently, Kimura [25,26] applied the Laplace–Carlson transforms (LCT) to price the American-style continuous-installment options. Ciurlia and Caperdori [12] extended the analysis to the perpetual continuous-installment case. Ciurlia [9–11] and Mezentsev et al. [28] provided alternative numerical methods to evaluate the European and American-style continuous-installment options. Finally, Yang et al. [33] and Yang and Yi [32] considered respectively a parabolic variational inequality problem arising from the European and American-style continuous-installment options and proved the existence and uniqueness of the solution to this problem and some properties of the free boundaries.

In all these paper mentioned above, the BS model is the starting point and in particular the volatility of underlying asset is assumed to be constant. However, the assumption of the constant volatility is in contrast to most of the empirical evidence of the option and stock pricing empirical literature which indicates that the stock's prices volatility is stochastic (see for example [18]). There are a number of stochastic volatility (SV) models have been developed as an extension to the BS model, such as [21,29,20], and many others (see [16] and the references therein for further details). In this paper we focus on Heston's stochastic volatility model, in which the variance uses a square root process.

Pricing American-style options under the SV model is a much more complicated task. Because of the early exercise feature of American-style option, an exact formula for the early exercise price is very difficult or even impossible to obtain, and research efforts have been focused on developing numerical approximation approaches which can price the American options accurately and faster than the lattice or simulation based methods. These approximations are based on integral representations of values of the American options (see, e.g., [24,22,2,23,31,8]). This paper we use the free surface series expansion of Tzavalis and Wang [31] to produce analytical approximations for the optimal stopping and exercise surfaces and price of this options. We derive these results using the Fourier inversion techniques, and then proceed to numerically solve the resulting non-linear integral equation systems for the free surfaces using the Newton–Raphson iteration procedure. Finally, we examine the impact that stochastic volatility has on the American continuous IOs relative to those priced under the BS model.

The outline of this paper is as follows. Section 2, we formulate the pricing problem for the American continuous-installment options under Heston SV model. Section 3 analyzes some properties of the optimal stopping and early exercise boundaries and provides general integral representations of the initial premiums for call and put case, each of which generates a system of nonlinear Volterra integral equations for the two free boundaries. Using the Fourier inversion transform and the series expansion of Tzavalis and Wang [31], we proceed to obtain an analytic integral equations for the call option price as well as the optimal stopping and early exercise boundaries in Section 4. The numerical algorithm for the implementation of a liked integral equation system for the two free boundaries are presented in Section 5, along with some numerical results including analysis of the impact that stochastic volatility has on option price and free boundaries. Section 6 concludes the paper.