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Blow-up criterion for the 3D compressible non-isentropic Navier–Stokes equations without thermal conductivity

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In this paper, we prove a blow-up criterion in terms of the density ρ and the pressure P for the strong solutions with vacuum to Cauchy problem of the 3D compressible non-isentropic Navier–Stokes equations without thermal conductivity. More precisely, we show that the strong solution exists globally if the norm $\|(\rho, P)\|_{L^\infty([0,T]\times\mathbb{R}^3)}$ is bounded from above.

1. Introduction

The motion of a viscous compressible fluid in $\mathbb{R}^3$ can be described by the compressible Navier–Stokes equations

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla P = \operatorname{div} \mathbb{T}, \\ (\rho e)_t + \operatorname{div}(\rho e u) + P \operatorname{div} u - \kappa \triangle \theta = \operatorname{div}(u\mathbb{T}) - u \operatorname{div} \mathbb{T}. \end{cases} \tag{1.1}$$

In this system, $x \in \mathbb{R}^3$ is the spatial coordinate, $t \geq 0$ is the time, ρ is the mass density, $u = (u^1, u^2, u^3) \in \mathbb{R}^3$ is the velocity vector of fluids, e is the specific internal energy, the constant κ is the thermal conductivity coefficient, P is the pressure satisfying

$$P = (\gamma - 1)\rho e = R\rho\theta, \tag{1.2}$$

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where θ is the absolute temperature, γ is the adiabatic exponent and R is a positive constant; $\mathbb{T}$ is the viscous stress tensor given by

$$\mathbb{T} = 2\mu D(u) + \lambda \operatorname{div} u \mathbb{I}_3, \tag{1.3}$$

where $\mathbb{I}_3$ is the 3×3 identity matrix, $D(u) = \frac{\nabla u + (\nabla u)^\top}{2}$ is the deformation tensor, μ is the shear viscosity coefficient, $\lambda + \frac{2}{3}\mu$ is the bulk viscosity coefficient, μ and λ are both real constants satisfying

$$\mu > 0, \qquad 3\lambda + 2\mu \geq 0, \tag{1.4}$$

which ensure the ellipticity of the Lamé operator L defined by

$$Lu = -\operatorname{div} \mathbb{T} = -\mu \triangle u - (\lambda + \mu)\nabla \operatorname{div} u. \tag{1.5}$$

When $\kappa = 0$, from (1.2), system (1.1) can be written as

$$\begin{cases} \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla P = \operatorname{div} \mathbb{T}, \\ P_t + u \cdot \nabla P + \gamma P \operatorname{div} u = (\gamma - 1)(\operatorname{div}(u\mathbb{T}) - u \operatorname{div} \mathbb{T}). \end{cases} \tag{1.6}$$

This paper is aimed at giving a blow-up criterion of strong solutions to the Cauchy problem of system (1.6) with the following initial data

$$(\rho, u, P)|_{t=0} = (\rho_0(x), u_0(x), P_0(x)), \quad x \in \mathbb{R}^3 \tag{1.7}$$

and the far field behavior

$$(\rho, u, P)(t, x) \to (0, 0, 0) \quad \text{as } |x| \to +\infty, \ t > 0. \tag{1.8}$$

Throughout this paper, we adopt the following simplified notations for the standard homogeneous and inhomogeneous Sobolev space:

$$\begin{aligned} &\|f\|_s = \|f\|_{H^s(\mathbb{R}^3)}, \qquad |f|_p = \|f\|_{L^p(\mathbb{R}^3)}, \qquad \|(f,g)\|_X = \|f\|_X + \|g\|_X, \\ &D^{k,r} = \{f \in L^1_{loc}(\mathbb{R}^3) : |\nabla^k f|_r < +\infty\}, \qquad D^k = D^{k,2}, \qquad |f|_{D^{k,r}} = \|f\|_{D^{k,r}(\mathbb{R}^3)}, \\ &D^1_0 = \{f \in L^6(\mathbb{R}^3) : |\nabla f|_2 < \infty\}, \qquad |f|_{D^1_0} = \|f\|_{D^1_0(\mathbb{R}^3)}. \end{aligned}$$

A detailed study on homogeneous Sobolev spaces can be found in [4].

As has been observed in Theorem 3 of Cho–Kim [3], in which the existence of unique local strong solution for system (1.6) was proved, in order to make sure that the Cauchy problem (1.6)–(1.8) with vacuum is well-posed, some compatibility condition on the initial data (ρ_0, u_0, P_0) was proposed to compensate the lack of a positive lower bound of the initial mass density ρ_0.

Theorem 1.1. *(See [3].) If the initial data (ρ_0, u_0, P_0) satisfy*

$$(\rho_0, P_0) \in H^1 \cap W^{1,q}, \qquad \rho_0 \geq 0, \qquad P_0 \geq 0, \qquad u_0 \in D^1_0 \cap D^2, \tag{1.9}$$

and the compatibility condition

$$Lu_0 + \nabla P_0 = \sqrt{\rho_0} h, \quad \textit{for some } h \in L^2, \tag{1.10}$$