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A linear programming formulation for constrained discounted continuous control for piecewise deterministic Markov processes ☆

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ABSTRACT

This paper deals with the constrained discounted control of piecewise deterministic Markov process (PDMPs) in general Borel spaces. The control variable acts on the jump rate and transition measure, and the goal is to minimize the total expected discounted cost, composed of positive running and boundary costs, while satisfying some constraints also in this form. The basic idea is, by using the special features of the PDMPs, to re-write the problem via an embedded discrete-time Markov chain associated to the PDMP and re-formulate the problem as an infinite dimensional linear programming (LP) problem, via the occupation measures associated to the discrete-time process. It is important to stress however that our new discrete-time problem is not in the same framework of a general constrained discrete-time Markov Decision Process and, due to that, some conditions are required to get the equivalence between the continuous-time problem and the LP formulation. We provide in the sequel sufficient conditions for the solvability of the associated LP problem, based on a generalization of Theorem 4.1 in [8]. In Appendix A we present the proof of this generalization which, we believe, is of interest on its own. The paper is concluded with some examples to illustrate the obtained results.

1. Introduction

Piecewise deterministic Markov processes (PDMPs) were introduced in [4] and [6] as a general family of continuous-time non-diffusion stochastic models, suitable for formulating many optimization problems in queuing and inventory systems, maintenance-replacement models, and many other areas of engineering

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and operations research. PDMPs are determined by three local characteristics: the flow ϕ, the jump rate λ, and the transition measure Q. Starting from x, the motion of the process follows the flow $\phi(x,t)$ until the first jump time T_1, which occurs either spontaneously in a Poisson-like fashion with rate λ or when the flow $\phi(x,t)$ hits the boundary of the state space. In either case the location of the process at the jump time T_1 is selected by the transition measure $Q(\phi(x,T_1),.)$ and the motion restarts from this new point as before. As shown in [6], a suitable choice of the state space and the local characteristics ϕ, λ, and Q provides stochastic models covering a great number of problems of engineering and operations research (see, for instance, [6,7]).

The objective of this work is to study the discounted continuous-time constrained optimal control problem of PDMPs by using the linear programming (LP) approach. Roughly speaking the formulation of the control problem is as follows. After each jump time, a control is chosen from a control set (which depends on the state variable) and will act on the jump rate λ and transition measure Q until the next jump time. The control variable will have two components, one that will parameterize a function that will regulate the jump rate and transition measure before the flow hits the boundary, and the other component that will act on the transition measure at the boundary (see Remark 2.2 below). The goal is to minimize the total expected discounted cost, which is composed of a running cost and a boundary cost (added to the total cost each time the PDMP touches the boundary). Both costs are assumed to be positive but not necessarily bounded. The constraints are also in the form of total expected discounted cost, again composed of positive running and boundary costs, not necessarily bounded. The state and control spaces are assumed to be general Borel spaces.

The linear programming technique has proved to be a very efficient method for solving continuous-time Markov Decision Processes (MDPs) with constraints. We do not attempt to present an exhaustive panorama on this topic, but instead we refer the interested reader to [1,2,14,13,16,19] and the references therein for detailed expositions on this technique in the context of continuous-time controlled Markov processes.

Contrary to continuous-time constrained MDPs, it is important to emphasize that constrained optimal control problems of PDMPs have received less attention. An attempt in this direction is presented in [12] where the authors study a control problem for a special class of PDMPs (with no boundary) by using an LP technique.

In this paper we aim at tracing a parallel with the theory developed for discounted discrete-time constrained MDP in general Borel spaces in order to get our results. We follow a similar approach as the one used for studying continuous-time MDPs without constraints, which consists of reducing the original continuous-time control problem into a semi-Markov or discrete-time MDP. For more details about such equivalence results, the reader may consult the recent survey [11] and the manuscript [10], and references therein. It is important to underline that such results cannot be directly applied in our case. Indeed, PDMPs are not piecewise constant processes and moreover, and more importantly, PDMPs have *deterministic jumps* in the sense that, roughly speaking, the process necessarily jumps when it hits the boundary. As a consequence, the inter-arrival jumping times are not exponentially distributed and the compensator of the process is not absolutely continuous as for continuous-time MDPs. Regarding PDMPs, the idea of reducing the original continuous-time control problem into a semi-Markov or discrete-time MDP was developed in [5] by reformulating the optimal control problem of a PDMP for a discounted cost as an equivalent discrete-time MDP in which the stages are the jump times T_n of the PDMP. In this paper we follow similar steps by re-writing the discounted continuous-time control constrained PDMP as a constrained discrete-time problem, in function of the post-jump location and control action, and with the stages being the jump times T_n of the PDMP. By doing this we can trace a parallel with the general theory for discrete-time MDPs in Borel spaces. As usual in these problems, we define the space of occupation measures associated to the discrete-time problem and, from this, we re-formulate the original problem as an infinite dimensional linear programming problem. It is important to stress however that our new discrete-time problem is given in terms of an MDP with an expected total cost criterion, in