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Periodic solutions for nonlinear differential inclusions with multivalued perturbations ☆

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ABSTRACT

In this paper, the periodic solutions for nonlinear differential inclusion governed by convex subdifferential and different perturbations are studied. It is firstly proved that the differential inclusion has unique periodic solution, if the perturbation function is a single-valued function. Then, by Schauder's fixed point theorem and Kakutani's fixed point theorem, we prove that the differential inclusion has at least a periodic solution, when the perturbation function is an upper semicontinuous (or lower semicontinuous) multifunction. Moreover, the existence of the extremal solution for the differential inclusion is also studied. Finally, based on one-sided Lipschitz (OSL) assumption, we prove the related relaxation theorem.

1. Introduction

Nonlinear differential equations arise in many fields of sciences (such as physics, mechanics, and material science), and have been studied by many authors in the last decades (see [1,3,6,9,18,21,23,24,26,27,29,32,37,40] and the references therein). Differential inclusions governed by subdifferential, which was first studied by Brezis in [6], play an important role in the theory of the nonlinear evolution equations (see [1,6,9,23,29,37,40]). In [6], Brezis studied existence theorem of the following differential inclusions governed by convex subdifferential

$$\begin{cases} \dot{x}(t) \in -\partial\varphi\big(x(t)\big) + f\big(t, x(t)\big) \\ x(0) = x_0 \end{cases} \tag{1}$$

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where $\partial\varphi(x)$ is the convex subdifferential of φ, and $f \in L^p([0,T],\mathbb{R}^n)$ is a single valued perturbation. Based on the conclusions in [6], using serval classical fixed point theorems, Attouch in [1] and Kravvaritis in [23] separately studied differential inclusions governed by subdifferential with multivalued perturbations as follows

$$\begin{cases} \dot{x}(t) \in -\partial\varphi\big(x(t)\big) + F\big(t, x(t)\big) \\ x(0) = x_0 \end{cases} \tag{2}$$

where $F(t, x(t))$ is a multi-valued perturbation. When φ is nonconvex, Qin and Xue in [29] studied existence theorem of the differential inclusions (2) governed by Clarke subdifferential. For more details about the problem (2), please see chapter II in [18].

As we know, periodic problem is motivated by physical problems. For example, the mathematical modeling of a variety of physical processes gives rise to periodic solutions. For this reason, existence of periodic solutions to nonlinear differential inclusions has been extensively investigated by many authors in the last decades (see [3,5,18,21,24,25,27,31,32,35]). Recently, periodic solution for (2) associated with monotone operators has received some attention (see [4,16,22,26,27,32,35]). For example, authors in [26] studied the existence of periodic solutions for the following nonlinear differential inclusion in $\mathbb{R}^m$,

$$\begin{cases} \dot{x}(t) \in -A(t)x(t) + F\big(t, x(t)\big) \\ x(0) = x(\omega) \end{cases} \tag{3}$$

where m is odd. Furthermore, authors in [27,35] considered the existence of periodic solutions for the problem (3) in Hilbert space and in general Banach spaces separately. And in [4], Bader and Papageorgiou studied the problem (3) with $A = \partial\varphi$ in Hilbert spaces. They all assumed a Nagumo type tangential condition in [4] when F satisfies a lower semi-continuity and an upper semicontinuity conditions. In [13], Filippakis and Papageorgiou studied the existence of solution of the differential inclusion (3) with $A = \partial\varphi$ in $\mathbb{R}^n$ and F is nonconvex.

In [41], under the one-sided Lipschitz condition, Xue and Yu studied the periodic problem for semi-linear differential inclusion of the form

$$\begin{cases} \dot{x}(t) \in -Dx(t) + F\big(t, x(t)\big) \\ x(0) = x(\omega). \end{cases} \tag{4}$$

Using Leray–Schauder alternative theorem, they obtained existence theorems for convex or nonconvex problem of (4). As an application, results in [41] can be used to prove the existence of periodic solutions of neural networks with discontinuous activations, which has been cited by many authors (see [20,30,38]). Meanwhile, in [41], they also studied "extremal solution" and relaxation theorem of (4). Extremal solution of differential equations plays an important role in control systems in connection with the "bang–bang" principle, and has been extensively studied (see [7,8,13,19,33]). However, the existence of extremal solution of (4), is far more difficult because $ext\, F(t,x)$ need not be closed and may not have any continuity properties, even if F is regular enough. In [13], authors studied the existence of extremal solution of the differential inclusion (3) with $A = \partial\varphi$ in $\mathbb{R}^n$. Moreover, in [19], Hu and Papageorgiou proved the existence of extremal solution and strong relaxation theorem of a nonlinear differential inclusion driven by a subdifferential operator.

Motivated by the above discussions, in this paper, we will study differential inclusions associated with convex subdifferential as follows

$$\begin{cases} \dot{x}(t) \in -Dx(t) - \partial\varphi\big(x(t)\big) + F\big(t, x(t)\big) \\ x(0) = x(\omega). \end{cases} \tag{5}$$