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Plane harmonic waves in strongly elliptic thermoelastic materials with microtemperatures

M. Ciarletta [a], F. Passarella [a], V. Tibullo [b],*

[a] *Dipartimento di Ingegneria Industriale, Università di Salerno, Italy*
[b] *Dipartimento di Ingegneria dell'Informazione, Ingegneria Elettrica e Matematica Applicata, Università di Salerno, Italy*

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ABSTRACT

This paper is concerned with the linear theory of thermoelasticity with microtemperatures, which permits the transmission of heat as thermal waves of finite speed, following the theory of Green and Naghdi. We study the propagation of plane harmonic waves in an unbounded strongly elliptic thermoelastic material with microtemperatures. We are able to prove that there is no dispersion and only undamped waves are possible. In the isotropic case we give the explicit analytic expression for the phase velocity of the waves. In particular we find that all transverse waves happen at constant temperature.

1. Introduction

During the past years, several theories concerning continuum mechanics characterized by a complex microstructure have been the object of intensive study (see [13,20] and references therein). There is also an increasing modern use of materials which possess thermal variations at microstructure level, as suspensions in viscous fluids, granular media, composites and nanomaterials. For example nanofluids, where very fine metallic oxide particles are held in suspension in a carrier fluid, are being increasingly employed in the heat transfer industry, see e.g. Vadasz et al. [40]. Other classes of nanomaterials are also being used and these typically show thermal effects at microstructure level. Also, cryogenic liquids have to be stored in vessels such as run tanks which store liquid hydrogen or oxygen for space research, see e.g. Jordan and Puri [27]. The stainless steel vessels which store such liquids require a better understanding of low-temperature behavior and operating limits.

Grot [17] developed the thermodynamic theory for elastic materials with inner thermal structure, according to which the molecules possess microtemperatures along with macro deformation of the body. In [36],

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* Corresponding author.
E-mail addresses: mciarletta@unisa.it (M. Ciarletta), fpassarella@unisa.it (F. Passarella), vtibullo@unisa.it (V. Tibullo).

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Riha studied the heat conduction in materials with microtemperatures and found that the experimental data for the silicon rubber containing spherical aluminum particles conform closely to predicted theoretical thermal conductivity. Further, Ieşan and Quintanilla [23] considered the simplest thermomechanical theory of elastic materials that takes into account the microtemperature variables and then they established some basic results concerning uniqueness, existence and asymptotic behavior of dynamic solutions and some equilibrium specific problems. Several papers based on this theory have been published (see, e.g. [6,19,21,39]).

On the other hand, due to experimental observations of thermal waves in solids, there is considerable interest in non-classical theories of thermoelasticity. In [22] Ieşan and Nappa have investigated the problem of heat flow in a micromorphic continuum with microtemperatures. They derived a new theory of heat for micromorphic continua which admits the possibility of *second sound* effects, by using balance laws established by Eringen [12] and the procedure proposed by Green and Naghdi [14–16]. Later, in the context of this theory, Ciarletta et al. [10] obtained continuous dependence on coupling coefficients and examined how the solution behaves as the microthermal coefficients terms tend to zero.

Moreover, in [24] Ieşan and Quintanilla established a thermomechanical theory of bodies with microstructure and microtemperatures. They presented a linearized theory, in contrast to that developed in [19], that permits the transmission of heat as thermal waves at finite speed. They obtained a uniqueness theorem and an instability result in the context of the special case of the theory, in which the stretch and the contraction of microelements is neglected.

In this paper, we study a homogeneous strongly elliptic thermoelastic medium with microtemperatures, following the theory developed by Ieşan and Quintanilla in [24]. For such a medium, we discuss the possibility of plane harmonic wave propagation. In [4] Chadwick explained the coupled character of the thermoelastic waves and noted that they are damped. Conversely, in the context of the present theory, we prove that there is neither dispersion nor attenuation in the wave propagation, and this is a direct consequence of the entropy balance proposed by Green and Naghdi [16] and the strong ellipticity condition; this is in contrast to what we see in [37] where the theory of thermoelasticity with microtemperatures of Ieşan and Quintanilla [23] is used. Further, we can see that thermal effects influence the propagation of thermoelastic waves in various contexts, see e.g. [1–3,5,9,11,25,26,35,38].

The results obtained, regarding the fact that there is no attenuation of energetic parameters of the thermoelastic wave, and there is not the dispersion phenomenon, are connected to the formulation of thermoelasticity of type II of Green and Naghdi. It is evident, also in view of applications, that in order to minimize the energy decay associated with the elastic propagation of the wave and to optimally control the phenomenon of dispersion, the thermoelastic process should have a temporal evolution such that the second law of thermodynamics should be as near as possible to the formulation of the GN theory recalled above. On the other hand, this application context is of considerable interest for auxetic materials, for which it is well known the coupling of high energy capacity absorptions with limited deformations.

The layout of the paper is a follows. In Section 2, we state the set of basic equations describing the behavior of thermoelastic media with microtemperatures within the context of the theory developed by [24] and we present the strong ellipticity condition. The ellipticity analysis is relevant in studying wave propagation [18] and has important applications in several contexts (see for examples, [8,28–30,32–34]). Moreover, we establish necessary and sufficient conditions characterizing the strong ellipticity for isotropic material. All results obtained under the hypothesis of strong ellipticity are thus valid also for classes of materials characterized by special properties, like negative Poisson's ratio and negative stiffness (auxetic or antirubber materials). These particular structures (see, for example, [7,31]) expand laterally when stretched, in contrast to the behavior of ordinary materials.

In Section 3, we seek for wave solutions to the basic differential system of the linear thermoelastodynamics under the strong ellipticity condition and prove that there exist only undamped plane harmonic waves for any direction of propagation. Further, in the isotropic case the possible waves are undamped in time and