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Global boundedness in a quasilinear chemotaxis system with signal-dependent sensitivity

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ABSTRACT

This paper deals with a quasilinear parabolic–elliptic chemotaxis system with signal-dependent sensitivity

$$\begin{cases} u_t = \nabla\cdot(\varphi(u)\nabla u) - \nabla\cdot(u\chi(v)\nabla v), & (x,t)\in\Omega\times(0,\infty), \\ 0 = \Delta v - v + u, & (x,t)\in\Omega\times(0,\infty), \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega\subset\mathbb{R}^n$ ($n\geq 2$), with nonnegative initial data $0\not\equiv u_0\in C^0(\overline{\Omega})$, where the given functions $\varphi(u)$ and $\chi(v)$ are the nonlinear diffusion and chemotactic sensitivity function, respectively. Firstly, under the case of non-degenerate diffusion $\varphi(u)$, it is proved that the corresponding initial boundary value problem possesses a unique global classical solution that is uniformly bounded in $\Omega\times(0,\infty)$. Moreover, under the case of degenerate diffusion $\varphi(u)$, we prove that the corresponding problem asserts at least one nonnegative global-in-time bounded weak solution.

1. Introduction

In this paper, we consider the following parabolic–elliptic Keller–Segel chemotaxis system with nonlinear diffusion and signal-dependent sensitivity under homogeneous Neumann boundary conditions

$$\begin{cases} u_t = \nabla\cdot(\varphi(u)\nabla u) - \nabla\cdot(u\chi(v)\nabla v), & (x,t)\in\Omega\times(0,\infty), \\ 0 = \Delta v - v + u, & (x,t)\in\Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & (x,t)\in\partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x), & x\in\Omega, \end{cases} \tag{1.1}$$

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where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a bounded domain with smooth boundary $\partial\Omega$, $\frac{\partial}{\partial\nu}$ denotes the differentiation with respect to the outward normal derivative on $\partial\Omega$, and $u(x,t)$, $v(x,t)$ denote the density of the cells population and the concentration of the chemoattractant, respectively. Moreover, the initial data $u_0 \in C^0(\bar{\Omega})$ is a given nonnegative function with $\int_\Omega u_0(x)dx > 0$, and the nonlinearities $\varphi(u)$ and $\chi(v)$ are supposed to be sufficiently smooth.

The oriented movement of biological cells or organisms in response to a chemical gradient is called *chemotaxis*. The most interesting situation related to self-organization phenomenon takes place when cells detect and response to a chemical which is secreted by themselves. The pioneering works of chemotaxis model were introduced by Patlak [19] in 1953 and Keller and Segel [15] in 1970, and we refer the reader to the survey [9,11,12] where a comprehensive information of further examples illustrating the outstanding biological relevance of chemotaxis can be found.

During the past decades, the chemotaxis models have become one of the best study models in numerous biological and ecological contexts, and the main issue of the investigation was whether the chemotaxis model allows for a chemotactic collapse, that is, if it possesses solutions that blow up in infinite or in finite time. However, in this paper, we will derive the uniform-in-time boundedness of problem (1.1) under some assumptions on the nonlinearities $\varphi(u)$, $\chi(v)$ and the initial data $u_0(x)$, which rules out the chemotactic collapse.

In order to better understand problem (1.1), let us mention some previous contributions in this direction. In recent years, the following initial boundary value problems have been studied by many authors

$$\begin{cases} u_t = \nabla\cdot(\varphi(u)\nabla u) - \nabla\cdot(u\chi(v)\nabla v), & (x,t) \in \Omega\times(0,\infty), \\ \tau v_t = \Delta v - v + u, & (x,t) \in \Omega\times(0,\infty), \\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = 0, & (x,t) \in \partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x), \tau v(x,0) = \tau v_0(x), & x \in \Omega, \end{cases} \tag{1.2}$$

where $\tau \in \{0,1\}$, $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$.

When $\tau = 1$, for the special case $\varphi(u) = 1$ in (1.2), Winkler [28] proved that problem (1.2) has a unique globally bounded classical solution provided that $0 < \chi(v) \leq \frac{\chi_0}{(1+\alpha v)^k}$ with some $\alpha > 0$ and $k > 1$ for any $\chi_0 > 0$. Recently, Fujie and Yokota [8] extended this result of [28] to the singular case $0 < \chi(v) \leq \frac{\chi_0}{v^k}$ for some $\chi_0 > 0$ and $k > 1$ under the assumptions on the initial data. Moreover, Zhang and Li [33] also generalized the result of [28] to a two-species chemotaxis system. Furthermore, when $\chi(v) = \frac{\chi_0}{v}$, the global existence of classical solutions to (1.2) is proved in [27] if $\chi_0 < \sqrt{\frac{2}{n}}$, moreover, if $\chi_0 < \sqrt{\frac{n+2}{3n-4}}$, then the global existence of weak solutions is established. Recently, Fujie [5] obtained the boundedness result for (1.2) with $\varphi(u) = 1$ and $\chi(v) = \frac{\chi_0}{v}$ under the case $0 < \chi_0 < \sqrt{\frac{2}{n}}$. In the radially symmetric setting, Stinner and Winkler [20] constructed certain weak solutions of (1.2) with $\chi(v) = \frac{\chi_0}{v}$ and $\chi_0 < \sqrt{\frac{n}{n-2}}$. Invoking additional dampening kinetic terms, Manásevich et al. [16] proved global existence and boundedness in a related system for any $\chi_0 > 0$ and some suitable initial data.

When $\tau = 0$, i.e., chemicals diffuse much faster than cells move (see [14]), for the special case $\varphi(u) = 1$, $\chi(v) = \frac{\chi_0}{v}$ in (1.2), Biler [2] proved the global existence of weak solutions under the condition $0 < \chi_0 < \frac{2}{n}$. Independently, Nagai and Senba [17] studied radially symmetric solutions to (1.2) with $\varphi(u) = 1$, $\chi(v) = \frac{\chi_0}{v}$, and they proved that solutions are globally bounded when either $n \geq 3$ and $0 < \chi_0 < \frac{n}{n-2}$ or $n = 2$ and $\chi_0 > 0$ is arbitrary, whereas if $n \geq 3$ and $\chi_0 > \frac{2n}{n-2}$, then there exist some finite-time blow-up solutions for (1.2). Without requiring such a symmetric case, for the cases $\varphi(u) = 1$ and $0 < \chi(v) \leq \frac{\chi_0}{v^k}$ in (1.2), Fujie et al. [6] proved that the corresponding problem possesses a unique globally bounded classical solution under some additional conditions on parameters χ_0 and k. Moreover, Fujie et al. [7] studied parabolic–elliptic problem (1.2) with $\varphi(u) = 1$, $\chi(v) = \frac{\chi_0}{v}$ invoking additional dampening kinetic terms, and proved that the