J. Math. Anal. Appl. 425 (2015) 111–132

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

On the rate of convergence to Rosenblatt-type distribution

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ARTICLE INFO

Article history:
Received 30 August 2013
Available online 12 December 2014
Submitted by M. Peligrad

Keywords:
Rate of convergence
Non-central limit theorems
Random field
Long-range dependence
Rosenblatt distribution
Generalized Linnik's covariance structure

ABSTRACT

The main result of the article is the rate of convergence to the Rosenblatt-type distributions in non-central limit theorems. Specifications of the main theorem are discussed for several scenarios. In particular, special attention is paid to the Cauchy, generalized Linnik's, and local–global distinguisher random processes and fields. Direct analytical methods are used to investigate the rate of convergence in the uniform metric.

1. Introduction

This paper studies local functionals of homogeneous random fields with long-range dependence, which appear in various applications in signal processing, geophysics, telecommunications, hydrology, etc. The reader can find more details about long-range dependent processes and fields in [7,10,12,16,37] and the references therein. In particular, [7] discusses different definitions of long-range dependence in terms of the autocorrelation function (the integral of the correlation function diverges) or the spectrum (the spectral density has a singularity at zero). The case when the summands/integrands are functionals of a long-range dependent Gaussian process is of great importance in the theory of limit theorems for sums/integrals of dependent random variables. It was shown by Taqqu [32,33] and Dobrushin and Major [6] that, comparing with the central limit theorem, long-range dependent summands can produce different normalizing coefficients and non-Gaussian limits. The volumes [7] and [28] give excellent surveys of the field. For multidimensional results of this type see [12,15,16]. Some most recent results can be found in [13,19,27].

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http://dx.doi.org/10.1016/j.jmaa.2014.12.016
0022-247X/

Despite recent progress in the non-central limit theory there has been remarkably little fundamental theoretical study on rates of convergence in non-central limit theorems. The rate of convergence to the Gaussian distribution for a local functional of Gaussian random fields with long-range dependence was first obtained in [15]. This result was applied to investigate the convergence of random solutions of the multi-dimensional Burgers equation in [21]. The only publications, which are known to the authors, on the rate of convergence to non-Gaussian distributions in the non-central limit theorem are [4,17]. These publications investigate particular cases of stochastic processes. The Hermite power variations of a discrete-time fractional Brownian motion were studied in [4]. The article [17] investigated the specific one-dimensional case of the Cauchy stochastic process and some facts used in the paper require corrections. To the best of our knowledge, the rate of convergence has never been studied in the general context of non-central limit theorems for non-Gaussian limit distributions. This work was intended as an attempt to obtain first results in this direction.

Our focus in this paper is on fine convergence properties of functionals of long-range dependent Gaussian fields. The paper establishes the rate of convergence in limit theorems for random fields, which is also new for the case of stochastic processes. It also generalizes the result of [17], which was obtained for a stochastic process with a fixed Cauchy covariance function, to integral functionals of random fields over arbitrary convex sets. In addition, the paper corrects some proofs in [17]. Specific important examples of the Cauchy, generalized Linnik, and local–global distinguisher random processes and fields, which have been recently used to separate a fractal dimension and the Hurst effect [10], are considered.

To estimate distances between distributions in the limit theorems for non-linear transformations of Gaussian stochastic processes Nourdin and Peccati proposed an approach based on the Malliavin calculus and Stein's method, see [23,24] and the references therein. The cases of the standard normal distribution and the centred Gamma distribution were considered and the limit theorems for the weakly dependent case were obtained. In [4] the Malliavin calculus and Stein's method were applied to obtain error bounds for Hermite power variations of a fractional Brownian motion. Central and non-central limit theorems for the Hermite variations of the anisotropic fractional Brownian sheet and the distance between a normal law and another law were studied in [30] and extended to the multidimensional case in [3]. However, to the best of our knowledge, there are no extensions of these results to more general classes of covariance functions in the multidimensional case considered in this paper. In contrast we use direct analytical probability methods to investigate the rate of convergence in the uniform (Kolmogorov) metric of long-range dependent random fields to the Rosenblatt-type distributions.

The class of Rosenblatt-type distributions is contained in the wide class of non-Gaussian Hermite distributions, which can be defined by its representation in the form of multiple Wiener–Itô stochastic integrals with respect to the complex Gaussian white noise random measure. The Rosenblatt distribution is a specific element from this class, which has been widely used recently in the probability theory and also appeared in a statistical context as the asymptotic distribution of certain estimators. There are power series expressions for the characteristic functions of the Rosenblatt distribution. For a comprehensive exposition of the Rosenblatt distribution and process we refer the reader to [9,20,32,34–36]. The approach presented in the present paper seems to be suitable even in more general situations of the Hermite limit distributions.

The results were obtained under assumptions similar to the standard ones in [31] and the references therein. Rather general assumptions were chosen to describe various asymptotic scenarios for correlation and spectral functions. Some simple sufficient conditions and examples of correlation models satisfying the assumptions are discussed in Sections 5 and 6.

As a bonus, some other new results of independent interest in the paper are: the boundedness of probability densities of the Rosenblatt-type distributions, asymptotics at the origin of the spectral densities of the Cauchy, generalized Linnik, and local–global distinguisher random processes and fields, and the representation of the spectral density of the local–global distinguisher random processes.