J. Math. Anal. Appl. 425 (2015) 133–159

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Homogenization for dislocation based gradient visco-plasticity ☆

Sergiy Nesenenko

Lehrstuhl für Nichtlineare Analysis und Modellierung, Fakultät für Mathematik, Universität Duisburg-Essen, Campus Essen, Thea-Leymann Strasse 9, 45117 Essen, Germany

ARTICLE INFO

Article history:
Received 9 May 2014
Available online 9 December 2014
Submitted by V. Radulescu

Keywords:
Gradient plasticity
Dislocations
Homogenization
Periodic unfolding
Korn's inequality
Rate-dependent models

ABSTRACT

In this work we study the homogenization for infinitesimal dislocation based gradient viscoplasticity with linear kinematic hardening and general non-associative monotone plastic flows. The constitutive equations in the models we study are assumed to be only of monotone type. Based on the generalized version of Korn's inequality for incompatible tensor fields (the non-symmetric plastic distortion) due to Neff/Pauly/Witch, we derive uniform estimates for the solutions of quasistatic initial-boundary value problems under consideration and then using a modified unfolding operator technique and a monotone operator method we obtain the homogenized system of equations. A new unfolding result for the Curl Curl-operator is presented in this work as well. The proof of the last result is based on the Helmholtz–Weyl decomposition for vector fields in general L^q-spaces.

1. Introduction

We study the homogenization of quasistatic initial-boundary value problems arising in gradient viscoplasticity. The models we study use rate-dependent constitutive equations with internal variables to describe the deformation behavior of metals at infinitesimally small strain.

Our focus is on a phenomenological model on the macroscale not including the case of single crystal plasticity. Our model has been first presented in [42]. It is inspired by the early work of Menzel and Steinmann [38]. Contrary to more classical strain gradient approaches, the model features from the outset a non-symmetric plastic distortion field $p \in \mathcal{M}^3$ [10], a dislocation based energy storage based solely on $|\operatorname{Curl} p|$ (and not ∇p) and therefore second gradients of the plastic distortion in the form of $\operatorname{Curl}\operatorname{Curl} p$ acting as dislocation based kinematical backstresses. We only consider energetic length scale effects and not higher gradients in the dissipation.

Uniqueness of classical solutions in the subdifferential case (associated plasticity) for rate-independent and rate-dependent formulations is shown in [41]. The existence question for the rate-independent model in

☆ Communicated with Patrizio Neff.
E-mail address: sergiy.nesenenko@uni-due.de.

http://dx.doi.org/10.1016/j.jmaa.2014.10.056

terms of a weak reformulation is addressed in [42]. The rate-independent model with isotropic hardening is treated in [21,42]. The well-posedness of a rate-dependent variant without isotropic hardening is presented in [49,50]. First numerical results for a simplified rate-independent irrotational formulation (no plastic spin, symmetric plastic distortion p) are presented in [46]. In [26,55] well-posedness for a rate-independent model of Gurtin and Anand [28] is shown under the decisive assumption that the plastic distortion is symmetric (the irrotational case), in which case one may really speak of a strain gradient plasticity model, since the full gradient acts on the symmetric plastic strain.

Let us shortly revisit the modeling ingredients of the gradient plasticity model under consideration. This part does not contain new results but is added for clarity of exposition. As usual in infinitesimal plasticity theory, the basic variables are the displacement $u : \Omega \to \mathbb{R}^3$ and the plastic distortion $p : \Omega \to \mathbb{R}^{3\times 3}$. We split the total displacement gradient ∇u into non-symmetric elastic and non-symmetric plastic distortions

$$\nabla u = e + p.$$

For invariance reasons, the elastic energy contribution may only depend on the symmetric elastic strains $\operatorname{sym} e = \operatorname{sym}(\nabla u - p)$. For more on the basic invariance questions related to this issue dictating this type of behavior, see [59,40]. We assume as well plastic incompressibility $\operatorname{tr} p = 0$, as is usual. The thermodynamic potential of our model is therefore written as

$$\int_\Omega \Big(\underbrace{\mathbb{C}[x]\big(\operatorname{sym}(\nabla u - p)\big)\big(\operatorname{sym}(\nabla u - p)\big)}_{\text{elastic energy}} + \underbrace{\frac{C_1[x]}{2}|\operatorname{dev}\operatorname{sym} p|^2}_{\text{kinematical hardening}} + \underbrace{\frac{C_2}{2}|\operatorname{Curl} p|^2}_{\text{dislocation storage}} + \underbrace{u\cdot b}_{\text{external volume forces}} \Big) dx \tag{1}$$

The positive definite elasticity tensor $\mathbb{C}$ is able to represent the elastic anisotropy of the material. The plastic flow has the form

$$\partial_t p \in g\big(\sigma - C_1[x]\operatorname{dev}\operatorname{sym} p - C_2 \operatorname{Curl}\operatorname{Curl} p\big), \tag{2}$$

where $\sigma = \mathbb{C}[x]\operatorname{sym}(\nabla u - p)$ is the elastic symmetric Cauchy stress of the material and g is a multivalued monotone flow function which is not necessary the subdifferential of a convex plastic potential (associative plasticity). This ensures the validity of the second law of thermodynamics, see [42].

In this generality, our formulation comprises certain non-associative plastic flows in which the yield condition and the flow direction are independent and governed by distinct functions. Moreover, the flow function g is supposed to induce a rate-dependent response as all materials are, in reality, rate-dependent.

Clearly, in the absence of energetic length scale effects (i.e. $C_2 = 0$), the $\operatorname{Curl}\operatorname{Curl} p$-term is absent. In general we assume that g maps symmetric tensors to symmetric tensors. Thus, for $C_2 = 0$ the plastic distortion remains always symmetric and the model reduces to a classical plasticity model. Therefore, the energetic length scale is solely responsible for the plastic spin (the non-symmetry of p) in the model.

Regarding the boundary conditions necessary for the formulation of the higher order theory we assume that the so-called micro-hard boundary condition (see [29]) is specified, namely

$$p \times n|_{\partial\Omega} = 0.$$

This is the correct boundary condition for tensor fields in L^2_{Curl}-spaces which admits tangential traces. We combine this with a new inequality extending Korn's inequality to incompatible tensor fields, namely