J. Math. Anal. Appl. 427 (2015) 629–645

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Asymptotic stability in linear viscoelasticity with supplies ☆

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A R T I C L E I N F O

Article history:
Received 28 May 2012
Available online 23 February 2015
Submitted by D.L. Russell

Keywords:
Asymptotic behavior
Linear viscoelasticity
Energy decay
Integro-differential equations

A B S T R A C T

We present some results about the asymptotic behavior of a linear viscoelastic system making use of the approach based on the concept of minimal state. This approach allows to obtain results in a larger class of solutions and data with respect to the classical one based on the histories of the deformation gradient. Recently, a lot of attention has been paid to find unified approaches which permit to study the asymptotic behavior with memory kernels presenting a temporal decay of which the exponential and polynomial decays are only special cases. Here we extend this unified approach to the dynamic problem in presence of supplies by using the minimal state and compare our results with those present in literature.

1. Introduction

The study of the asymptotic behavior of viscoelastic materials has been focused almost exclusively on the study of the stability in the topology defined on the histories space of the Graffi–Volterra free energy and with zero data both for the initial history and for the external source.

In this research we will study the stability and the asymptotic behavior of the energy in the states space, just as defined by Banfi [5] and then proposed again by Del Piero and Deseri [11] (see Appendix A). We will use the topology induced by the free energy introduced in [13] which allows to work in a wider space of states with respect to the one defined through the Graffi–Volterra free energy [6].

In presence of memory kernels of exponential type, many results concerning the exponential stability of the Graffi–Volterra free energy and of the one proposed in [13] have been obtained ([4,14,15] and references therein) starting from the pioneering paper by Dafermos [10]; while, if the memory kernel polynomially decays in time, long time behavior results have been obtained for vanishing data [20].

☆ Research performed under the auspices of G.N.F.M. – I.N.d.A.M. and partially supported by Italian M.I.U.R.

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http://dx.doi.org/10.1016/j.jmaa.2015.02.061
0022-247X/

Here, as done in [16] for a viscoelastic fluid, we will consider memory kernels presenting a temporal decay from which we can recover the exponential decay or the polynomial decay as special cases (see [19]) and prove that the decay of the solutions is strictly related to the behavior of the memory kernel.

Another innovative aspect of this research lies in the possibility of studying such problems in presence of non-zero past histories and external sources. We will compare our results with those obtained in a recent paper by Guesmia [18], which uses an adaptation of the so-called convexity method introduced in [1] within the context of the topology induced by the Graffi–Volterra free energy and with no supply.

This paper is organized as follows. In Section 2, the differential system is formulated as an initial boundary value problem, but also within the semigroup theory. Hence, we recall some results about its well posedness. In Section 3 we present our results on the asymptotic behavior and exponential stability. In Section 4, we give some general comments and state some open problems. Finally, in Appendix A, we recall the classical constitutive equation of the viscoelasticity and the notion of minimal state.

2. Setup of the problem

The initial boundary value problem, for a standard linear viscoelastic problem in a bounded domain $\Omega \subset \mathcal{R}^3$ with regular boundary $\partial\Omega$, is governed by the equation (see Appendix A)

$$\frac{\partial}{\partial t} v(x,t) = \nabla \cdot \left[g_\infty \mathbb{A} \nabla u(x,t) + \int_0^\infty g'(s) \mathbb{A} \nabla u_r^t(x,s)\, \mathrm{d}s \right] + f(x,t), \tag{2.1}$$

where the vector v denotes the velocity, the vector u the displacement, the function $u_r^t(\cdot,s) = u(\cdot,t-s) - u(\cdot,t)$ the relative past history of u, while the positive constant g_∞ and the function $\breve{g}(s) = -\int_s^\infty g'(s)\mathrm{d}s$ satisfy the following conditions:

(h$_1$) $g_\infty > 0$, g' and $\breve{g}$ belong to $L^1(\mathcal{R}^+)$,
(h$_2$) $\breve{g}$ is positive in $\mathcal{R}^+$ and

$$\int_0^\infty \breve{g}(s) \cos \omega s \, \mathrm{d}s > 0, \quad \forall \omega \in \mathcal{R}.$$

Moreover, the fourth order tensor $\mathbb{A}$ is symmetric and positive definite, the density ρ is assumed equal to 1 and f denotes the body force.

Together with Eq. (2.1), initial and boundary conditions are given by

$$\begin{aligned} &v(x,0) = v_0(x),\ u(x,0) = u_0(x),\ u^{t=0}(x,s) = u^0(x,s),\ s > 0, x \in \Omega, \\ &u(x,t) = 0, \quad x \in \partial\Omega,\ t \in \mathcal{R}^+. \end{aligned} \tag{2.2}$$

We observe that the following equalities hold:

$$\int_t^\infty g'(s) \mathbb{A} \nabla u_r^t(x,s)\, \mathrm{d}s = \int_0^\infty g'(t+s) \mathbb{A} \nabla u_r^{t=0}(x,s)\, \mathrm{d}s + \breve{g}(t) \mathbb{A} \nabla [u(x,t) - u(x,0)],$$

$$\int_0^t g'(s) \mathbb{A} \nabla u_r^t(x,s)\, \mathrm{d}s = [\breve{g}(t) + g_\infty] \mathbb{A} \nabla [u(x,0) - u(x,t)] + \int_0^t [\breve{g}(t-s) + g_\infty] \mathbb{A} \nabla v(x,s)\, \mathrm{d}s,$$

so that Eq. (2.1) becomes