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Existence of global solutions for isentropic gas flow in a divergent nozzle with friction

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ABSTRACT

We study isentropic gas flow in a divergent nozzle with friction. It is physically known that the flow is influenced by friction. In this paper, we investigate the mathematical effect of the friction on solutions. We first construct approximate solutions by using the Godunov scheme and the fractional step procedure. We obtain the bounded estimate by the invariant region. This is the difficult point of the present paper. We use the compensated compactness to prove their convergence. Finally, we prove the existence of global solutions with arbitrary L^∞ data.

1. Introduction

The present paper is concerned with isentropic gas flow in a divergent nozzle. In addition, we consider friction due to viscosity (see [12, Section 6]). This motion is governed by the following equations:

$$\begin{cases} \rho_t + m_x = -a(x)m, \\ m_t + \left(\dfrac{m^2}{\rho} + p(\rho)\right)_x = -a(x)\dfrac{m^2}{\rho} - \alpha\dfrac{m|m|}{\rho}, \quad x \in \mathbf{R}, \end{cases} \tag{1.1}$$

where ρ, m and p are the density, the momentum and the pressure of gas, respectively. If $\rho > 0$, $v = m/\rho$ represents the velocity. For the barotropic gas, $p(\rho) = \rho^\gamma/\gamma$, where $\gamma \in (1, 5/3]$ is the adiabatic exponent for usual gases. Moreover, we denote a friction constant by $\alpha > 0$. The given function $a(x)$ is represented by

$$a(x) = A'(x)/A(x) \quad \text{with } A(x) = e^{\int^x a(y)dy},$$

where $A \in C^1(\mathbf{R})$ is a slowly variable cross section area at x in the nozzle. Since we consider the divergent nozzle, we assume that $A'(x) \geq 0$.

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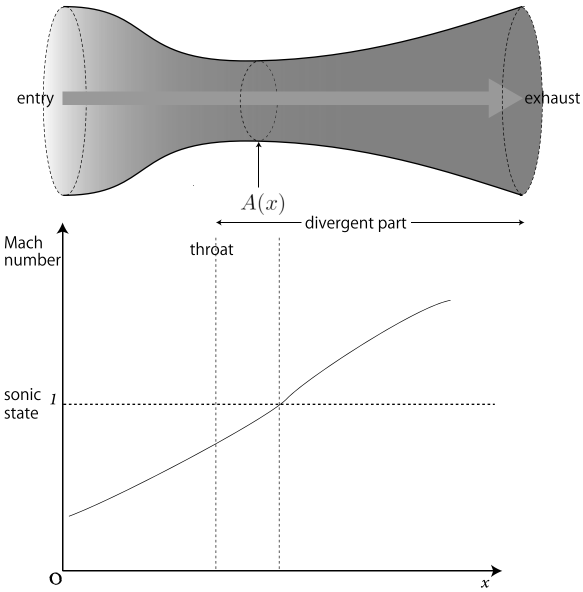

Fig. 1. Laval nozzle and Mach number.

Then, we consider the Cauchy problem (1.1) with

$$(\rho, m)|_{t=0} = \big(\rho_0(x), m_0(x)\big). \tag{1.2}$$

Setting $u = {}^t(\rho, m)$, the above problem (1.1)–(1.2) can be written in a compact form as

$$\begin{cases} u_t + f(u)_x = g(x,u), \quad x \in \mathbf{R}, \\ u|_{t=0} = u_0(x), \end{cases} \tag{1.3}$$

where $f(u) = {}^t(m, \frac{m^2}{\rho} + p(\rho))$, $g(x,u) = {}^t(-a(x)m, -a(x)\frac{m^2}{\rho} - \alpha\frac{m|m|}{\rho})$.

We survey the related results. For one-dimensional case, in [5], DiPerna proved the global existence by the vanishing viscosity method and a compensated compactness argument. DiPerna applied the method to systems first, for the special case where $\gamma = 1 + 2/n$ and n is an odd integer. Subsequently, Ding, Chen and Luo extended the analysis to any γ in $(1, 5/3]$ (see [1] and [3]). Moreover, in [4], they studied isentropic gas dynamics with inhomogeneous terms by the fractional step procedure. Their method shall be used in the present paper.

For nozzle flow without friction, in [6] and [10], Lu and Gu investigated the nozzle flow with the monotone cross section. In [14], the present author treated with the Laval nozzle, the convergent and divergent nozzle. The Laval nozzle is the most important type of nozzles. The nozzle is a tube that is pinched in the middle, making an hourglass-shape (see Fig. 1). Moreover, in [15], the same author studied the general nozzle.

Now, friction has influence on the nozzle flow (see [12, Chapter 6]). For the Laval nozzle, the flow attains the sonic state at the throat, the point where the cross section is minimum (see Fig. 1 and [9]). However, friction moves the point to the downstream. In this paper, we investigate properties of solutions in the divergent part of the Laval nozzle after the flow attains the sonic state.

To state our main theorem, we define the Riemann invariants w, z, which play important roles in this paper, as

$$z := \frac{m}{\rho} - \frac{\rho^\theta}{\theta} = v - \frac{\rho^\theta}{\theta}, \qquad w := \frac{m}{\rho} + \frac{\rho^\theta}{\theta} = v + \frac{\rho^\theta}{\theta} \quad \left(\theta := \frac{\gamma - 1}{2}\right). \tag{1.4}$$