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Hyers–Ulam stability of linear functional differential equations

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A B S T R A C T

In this paper, the stability of some classes of linear functional differential equations was discussed by direct method, iteration method, fixed point method and open mapping theorem. It is shown that the Hyers–Ulam stability holds true for $y^{(n)} = g(t)y(t-\tau) + h(t)$. The stability of functional differential equations with multiple delays of first order and general delay differential equations also have been discussed.
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1. Introduction and preliminaries

In 1940, S.M. Ulam [26] raised a problem whether an approximate solution of a functional equation can be approximated by a solution of the corresponding equation. This question was first answered by D.H. Hyers [13] one year later. Thereafter, T. Aoki [2], D.G. Bourgin [4] and Th.M. Rassias [23] improved the result of D.H. Hyers. For more details and further discussions, we refer the readers to the book by S.-M. Jung [17]. In recent years, accompanied by the development of the Hyers–Ulam stability of functional equations (see, e.g. [5–7]), many mathematicians paid attention to the problem of differential equations and for the papers concerning the stability of ordinary differential equations we refer to [14,18–21]. Further, the interested readers can see [1,9,15] concerning the stability of partial differential equations.

Delay differential equations (DDEs) are a special type of ordinary differential equations in which the derivative of the unknown function at a certain time is given in terms of the values of the function at previous times (advanced differential equations (ADEs) for the case of future times), which are very important in life science and engineering. DDEs are also called functional differential equations (FDEs) with delays [27].

To the best of our knowledge, the first mathematicians who investigated the stability of DDEs are S.-M. Jung and J. Brzdęk [18]. They proved the Hyers–Ulam stability of $y'(t) = \lambda y(t-\tau)$ with an initial condition, where $\lambda > 0$ and $\tau > 0$ are real constants. Motivated by their work, we investigate the Hyers–Ulam

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stability of more general linear FDEs with constant delay(s) on a bounded interval by some other methods. (The case for advanced functional differential equations is absolutely the same.)

$\Gamma^n[a,b;\tau]$ denotes the family of all continuous functions mapping the real interval $[a-\tau,b]$ into $\mathbb{R}$ which are n times continuously differentiable on $[a,b]$. Without loss of generality, we focus our attention on $\Gamma^n[0,b;\tau]$ in this paper.

2. Stability of $y^{(n)} = g(t)y(t-\tau) + h(t)$

In this section, we study the Hyers–Ulam stability of delay differential equations of the form $y^{(n)} = g(t)y(t-\tau) + h(t)$ on $[0,\tau]$ by some different methods, where $\tau > 0$ is a constant.

Now, we spend some time presenting in details some possibilities for the proof of Hyers–Ulam stability of $y^{(n)} = g(t)y(t-\tau) + h(t)$, to see the advantages and the disadvantages.

2.1. Open mapping theorem method

In 2003, H. Takagi, T. Miura and S.-E. Takahasi [25] showed that open mapping theorem is closely linked with the Hyers–Ulam stability, which makes the stability problem of linear bounded operator much more explicit.

We need some terminologies before displaying their result. Let $\ker T$ denote the kernel of the bounded linear operator $T : A \to B$. Define the induced one-to-one operator $\tilde{T}$ from the quotient space $A/\ker T$ into B by

$$\tilde{T}(u + \ker T) := Tu \quad \forall u \in A.$$

Definition 2.1. We say an operator T has the Hyers–Ulam stability, if there is a constant K with the following properties:

For any $g \in T(A)$, $\varepsilon \geq 0$ and $f \in A$ satisfying $\|Tf - g\| \leq \varepsilon$, there exists $f_0 \in A$ such that $Tf_0 = g$ and $\|f - f_0\| \leq K\varepsilon$.

We call K a Hyers–Ulam stability constant of operator T.

Definition 2.2. We say an equation $Tf = g$ has the Hyers–Ulam stability, if there is a constant K with the following properties:

For any $\varepsilon \geq 0$ and $f \in A$ satisfying $\|Tf - g\| \leq \varepsilon$, there exists $f_0 \in A$ such that $Tf_0 = g$ and $\|f - f_0\| \leq K\varepsilon$.

We call K a Hyers–Ulam stability constant of equation $Tf = g$.

Remark 2.3. Obviously, if operator T has the Hyers–Ulam stability and $g \in T(A)$, then equation $Tf = g$ has the Hyers–Ulam stability.

Theorem 2.4. *Let A and B be Banach spaces and T be a bounded linear operator from A into B. Then the following statements are equivalent:*

(a) T has the Hyers–Ulam stability;
(b) The range of T is closed;
(c) $\tilde{T}^{-1}$ is a bounded linear operator.

Moreover, if one of the conditions is true, then the infimum of the set of Hyers–Ulam stability constants is $\|\tilde{T}^{-1}\|$.

Next, we give the definition of the Hyers–Ulam stability of Eq. (1) in the classic sense.

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