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Products of composition, multiplication and radial derivative operators from logarithmic Bloch spaces to weighted-type spaces on the unit ball

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ABSTRACT

Let $H(\mathbb{B})$ denote the space of all holomorphic functions on the unit ball $\mathbb{B}$ of $\mathbb{C}^n$, $\psi \in H(\mathbb{B})$ and φ be a holomorphic self-map of $\mathbb{B}$. Let C_φ, M_ψ and $\mathcal{R}$ denote the composition, multiplication and radial derivative operators, respectively. In this paper, we characterize the boundedness and compactness of linear operators induced by products of these operators from logarithmic Bloch spaces to weighted-type spaces on the unit ball.

1. Introduction

Let $z = (z_1, \cdots, z_n)$ and $w = (w_1, \cdots, w_n)$ be points in the complex vector space $\mathbb{C}^n$ and $z\overline{w} := \langle z, w\rangle = z_1\overline{w_1} + z_2\overline{w_2} + \cdots + z_n\overline{w_n}$ the inner product of z and w, where $\overline{w_k}$ is the complex conjugate of w_k. We also write

$$|z| = \sqrt{\langle z, z\rangle} = \sqrt{|z_1|^2 + |z_2|^2 + \cdots + |z_n|^2}.$$

Let $\mathbb{B} = \{z \in \mathbb{C}^n : |z| < 1\}$ be the open unit ball in $\mathbb{C}^n$, $H(\mathbb{B})$ the class of all holomorphic functions on the unit ball. Let $\psi \in H(\mathbb{B})$ and φ be a holomorphic self-map of $\mathbb{B}$. Composition, multiplication and radial derivative operators on $H(\mathbb{B})$ were defined as follows:

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$$(C_\varphi f)(z) = (f \circ \varphi)(z) = f\big(\varphi(z)\big), \quad z \in \mathbb{B};$$
$$(M_\psi f)(z) = \psi(z) f(z), \quad z \in \mathbb{B};$$
$$\mathcal{R} f(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z), \quad z \in \mathbb{B}.$$

It is well known that [13,37]

$$\mathcal{R} f(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z) = \langle \nabla f(z), \bar{z} \rangle,$$

where

$$\nabla f(z) = \left(\frac{\partial f}{\partial z_1}(z), \frac{\partial f}{\partial z_2}(z), \cdots, \frac{\partial f}{\partial z_n}(z) \right)$$

is the complex gradient of function f.

Let $\varphi = (\varphi_1, \varphi_2, \cdots, \varphi_n)$ denote a holomorphic self-map of $\mathbb{B}$. Write

$$\mathcal{R}\varphi(z) = \big(\mathcal{R}\varphi_1(z), \mathcal{R}\varphi_2(z), \cdots, \mathcal{R}\varphi_n(z)\big),$$
$$\big|\varphi(z)\big| = \sqrt{\sum_{j=1}^{n} \big|\varphi_j(z)\big|^2},$$
$$\big|\mathcal{R}\varphi(z)\big| = \sqrt{\sum_{j=1}^{n} \big|\mathcal{R}\varphi_j(z)\big|^2}.$$

For $\psi_1, \psi_2, \psi_3 \in H(\mathbb{B})$, we introduce the following operator

$$T_{\psi_1,\psi_2,\psi_3,\varphi} f(z) = \psi_1(z) f\big(\varphi(z)\big) + \psi_2(z) \mathcal{R} f\big(\varphi(z)\big) + \psi_3(z) \mathcal{R}(f \circ \varphi)(z), \quad f \in H(\mathbb{B}).$$

It is clear that all products of composition, multiplication and radial derivative operators in the following six ways can be obtained from the operator $T_{\psi_1,\psi_2,\psi_3,\varphi}$ by fixing ψ_1, ψ_2, ψ_3. More specifically we have

$$M_\psi C_\varphi \mathcal{R} = T_{0,\psi,0,\varphi}, \qquad C_\varphi \mathcal{R} M_\psi = T_{\mathcal{R}\psi(\varphi),\psi(\varphi),0,\varphi}, \qquad C_\varphi M_\psi \mathcal{R} = T_{0,\psi\circ\varphi,0,\varphi},$$
$$\mathcal{R} M_\psi C_\varphi = T_{\mathcal{R}\psi,0,\psi,\varphi}, \qquad M_\psi \mathcal{R} C_\varphi = T_{0,0,\psi,\varphi}, \qquad \mathcal{R} C_\varphi M_\psi = T_{\mathcal{R}(\psi\circ\varphi),0,\psi\circ\varphi,\varphi}.$$

The logarithmic Bloch space $\mathcal{B}_{\log}$ consists of all $f \in H(\mathbb{B})$ such that [17]

$$\|f\| = \sup_{z\in\mathbb{B}} \big(1 - |z|^2\big) \left(\log \frac{e}{1 - |z|^2} \right) \big|\mathcal{R} f(z)\big| < \infty.$$

The little logarithmic Bloch space $\mathcal{B}_{\log,0}$ consists of all $f \in H(\mathbb{B})$ satisfying

$$\lim_{|z|\to 1} \big(1 - |z|^2\big) \left(\log \frac{e}{1 - |z|^2} \right) \big|\mathcal{R} f(z)\big| = 0.$$

It is easy to see that both $\mathcal{B}_{\log}$ and $\mathcal{B}_{\log,0}$ are Banach spaces with the norm

$$\|f\|_{\mathcal{B}_{\log}} = \big|f(0)\big| + \|f\|.$$