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Invasion by an inferior or superior competitor: A diffusive competition model with a free boundary in a heterogeneous environment

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A B S T R A C T

In this study, we consider the population dynamics of an invasive species and a resident species, which are modeled as a diffusive competition process in a radially symmetric setting with a free boundary. We assume that the resident species undergoes diffusion and growth in $\mathbb{R}^n$, while the invasive species initially exists in a finite ball, but invades the environment with a spreading front evolving according to a free boundary. When the invasive species is inferior, we show that if the resident species is already well established initially, then the invader can never invade deep into the underlying habitat, thus it dies out before its invading front reaches a certain finite limiting position. When the invasive species is superior, a spreading–vanishing dichotomy holds, and sharp criteria for spreading and vanishing with d_1, μ, and u_0 as variable factors are obtained, where d_1, μ, and u_0 are the dispersal rate, expansion capacity, and initial number of invaders, respectively. In particular, we obtain some rough estimates of the asymptotic spreading speed when spreading occurs.

1. Introduction

Various models are used to describe competition and co-existence dynamics in population ecology. A typical example is the following Lotka–Volterra competition reaction–diffusion system for two species in a bounded smooth domain $\Omega \subset \mathbb{R}^n$:

$$\begin{cases} u_t - d_1\Delta u = \big[a_1(x) - b_1(x)u - c_1(x)v\big]u, & x \in \Omega,\ t > 0, \\ v_t - d_2\Delta v = \big[a_2(x) - b_2(x)u - c_2(x)v\big]v, & x \in \Omega,\ t > 0, \\ u(x,t) = v(x,t) = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) \ge 0, \quad v(x,0) = v_0(x) \ge 0, & x \in \Omega, \end{cases} \tag{1.1}$$

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where u and v denote the population densities of two competing species; d_1 and d_2 are positive and they represent the dispersal rates of two species; and the strictly positive functions $a_i(x)$, $b_i(x)$, and $c_i(x)$ ($i = 1, 2) \in C^1(\Omega) \cap L^\infty(\overline{\Omega})$ denote the local growth rate of the population or the density of the local resource, the self-regulation of species, and competition between species, respectively.

In general, the long-term dynamics comprise one of the main problems investigated using (1.1) and they are quite well understood. The reader may refer to [4,5,14] and the references therein for further details. Typically, previous studies suggest that weak competition allows the coexistence of states in (1.1), whereas stronger competition leads to the extinction of species with low reproduction rates and large diffusion rates. More precisely, let λ^* be the principal eigenvalue of the operator $-\Delta$ in Ω subject to the homogeneous Dirichlet boundary conditions. Set

$$a_{iL} = \inf_{x\in\overline{\Omega}} a_i(x), \qquad a_{iM} = \sup_{x\in\overline{\Omega}} a_i(x)$$

for $i = 1, 2$, and b_{iL}, b_{iM}, c_{iL}, and c_{iM} ($i = 1, 2$) are defined analogously. Then, the following results have been proved for (1.1):

1. If $a_{1L} > d_1\lambda^* + \frac{a_{2M}c_{1M}}{c_{2L}}$ and $a_{2L} > d_2\lambda^* + \frac{a_{1M}b_{2M}}{b_{1L}}$, then a coexistence state exists for (1.1), i.e., a stationary solution (u^*, v^*) with $u^*, v^* > 0$ in Ω;
2. If $\frac{a_{1M}}{a_{2L}} < \min\{\frac{c_{1L}}{c_{2M}}, \frac{b_{1L}}{b_{2M}}\}$, $a_{2L} \geq a_{1M}$, $d_1 = d_2 = D$, and $a_{2L} > D\lambda^*$, then the species u will eventually be driven to extinction, i.e., $\lim_{t\to\infty} u(x,t) = 0$ for any $v_0 \not\equiv 0$;
3. If $\frac{a_{1L}}{a_{2M}} > \max\{\frac{c_{1M}}{c_{2L}}, \frac{b_{1M}}{b_{2L}}\}$, $a_{1L} \geq a_{2M}$, $d_1 = d_2 = D$, and $a_{1L} > D\lambda^*$, then the species v is eventually driven to extinction, i.e., $\lim_{t\to\infty} v(x,t) = 0$ for any $u_0 \not\equiv 0$.

In the second case, the competitor u is wiped out by v in the long term and v will win the competition in an ecological context, thus we refer to v as the superior competitor and u as the inferior competitor. Analogously, v is the inferior competitor and u is the superior competitor in the third case. The first case is often regarded as the weak competition case, where neither competitor wins or loses the competition.

However, we still note that the model (1.1) is not realistic for describing the dynamics of a new competitive species that invades the habitat of a resident species because of the limited fixed domain and the lack of information about the precise invasion dynamics. Thus, it is necessary to relax these requirements and to consider the precise dynamics of an invading species when it spreads in a new habitat.

Given this aim, the current study is concerned with the impact of spatial features in an environment on the dynamics of a new competitor u with a free boundary to describe the moving front when it invades the habitat of a resident species v. For simplicity, we assume that the environment is radially symmetric and we investigate the behavior of the positive solution $(u(r,t), v(r,t), h(t))$ with $r := |x|$ ($x \in \mathbb{R}^n$) to the following variation of the reaction–diffusion problem (1.1):

$$\begin{cases} u_t - d_1\Delta u = \big[a_1(r) - b_1(r)u - c_1(r)v\big]u, & 0 < r < h(t),\ t > 0, \\ v_t - d_2\Delta v = \big[a_2(r) - b_2(r)u - c_2(r)v\big]v, & 0 < r < \infty,\ t > 0, \\ u_r(0,t) = v_r(0,t) = 0, \quad u(r,t) = 0, & h(t) \leq r < \infty,\ t > 0, \\ h'(t) = -\mu u_r\big(h(t),t\big), & t > 0, \\ u(r,0) = u_0(r), \quad h(0) = h_0, & 0 \leq r \leq h_0, \\ v(r,0) = v_0(r), & 0 \leq r < \infty, \end{cases} \tag{1.2}$$

where $\Delta u = u_{rr} + \frac{n-1}{r}u_r$, $r = h(t)$, denotes the spreading front, i.e., the free boundary that needs to be determined; $d_1, d_2 > 0$ are diffusion rates; $\mu > 0$, the expansion capacity, is the ratio of the expansion speed of the free boundary relative to the population gradient at the expanding front, which explains the ability