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Well-posedness for a model derived from an attraction–repulsion chemotaxis system

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ABSTRACT

In this paper, we are interested in a model derived from an attraction–repulsion chemotaxis system in high dimensions:

$$\begin{cases} \partial_t u - \Delta u = -\beta_1 \nabla \cdot (u \nabla v) + \beta_2 \nabla \cdot (u \nabla w), & x \in \mathbb{R}^n,\ t > 0, \\ \lambda_1 v - \Delta v = u, & x \in \mathbb{R}^n,\ t > 0, \\ \lambda_2 w - \Delta w = u, & x \in \mathbb{R}^n,\ t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^n, \end{cases}$$

with the parameters $\beta_1 \geq 0$, $\beta_2 \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ and nonnegative initial data $u_0(x) \in L^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n)$. We prove that a global bounded solution exists when the repulsion prevails over the attraction in the sense of $\beta_1 < \beta_1$. Moreover, we give the smoothness of the solution and obtain its decay rates in $W^{s,p}(\mathbb{R}^n)$, which coincide with the ones of the classical heat equation. Conversely, when $n = 2$, $\beta_1 > \beta_2$, we prove that the finite time blow-up may occur.

1. Introduction

Chemotaxis is a phenomenon describing the movements of cells, bacteria or organisms in response to some chemical substances. According to the orientation of the movements, we can describe the phenomenon by positive (chemoattractive) chemotaxis and negative (chemorepulsive) chemotaxis, respectively. The former occurs when the movement is toward a higher concentration of the chemical, and conversely, the latter occurs when the movement is in the opposite direction. In both of cases, the chemical substances are called chemoattractants and chemorepellents, respectively, which play a crucial role in many developmental systems. In recent years, the chemotaxis phenomena have been attracting more and more researchers to study in various views. That's because it has various interesting issues, such as global solvability, finite time blow-up, time asymptotic behavior and nontrivial stationary solutions, a very important one of which is the

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blow-up phenomenon. And moreover, in research of the chemotaxis phenomena, mathematical analysis is playing a more and more important role and is very helpful in understanding the phenomena.

A classical chemotaxis model is the well-known Keller–Segel (KS) model [9] proposed by Keller and Segel in 1970s. It can be described as:

$$\begin{cases} \partial_t u - \Delta u = -\nabla \cdot (\beta u \nabla v), & x \in \Omega,\ t > 0, \\ \kappa v_t - \Delta v - u + \lambda v = 0, & x \in \Omega,\ t > 0, \end{cases} \tag{1.1}$$

where $u = u(x,t)$ denotes the density of cells and $v = v(x,t)$ denotes the concentration of the chemoattractant; $\lambda > 0$ is a constant and $\beta > 0$ represents the chemoattractant sensitivity, and the domain $\Omega \in \mathbb{R}^n$. If Ω is a bounded domain with smooth boundary, we can propose the homogeneous Neumann initial boundary value conditions

$$\begin{cases} \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.2}$$

and if $\Omega = \mathbb{R}^n$, we have the initial data

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}^n. \tag{1.3}$$

One key feature of the system is the competition between diffusion and nonlocal aggregation, and, as a result, there are many consequences mainly about blow-up phenomenon and global existence of classical solutions. Here we try our best knowledge to simply summarize these results in the following.

For initial boundary value problem: When $n = 1$, the solution to the problem (1.1) and (1.2) exists globally and uniformly bounded (see [13,19]), whereas, when $n \geq 2$, the blow-up may occur. For $\kappa = 1$, M. Winkler [25] proved that the solution of (1.1) and (1.2) in $\mathbb{R}^n$ ($n \geq 3$) may blow up either in finite or infinite time even if the initial mass $\int_\Omega u_0(x)dx$ is small, and when $n = 2$, there is a global and bounded solution if the initial mass is less than some critical mass (see [5,16]) and there is finite or infinite time blow-up if the initial mass is larger than some critical mass (see [8,21]). For the case $\kappa = 0$ and $n \geq 2$, T. Nagai [13,15] showed that the finite time blow-up occurs if $\int_\Omega u_0|x|^n dx$ is sufficiently small and $\int_\Omega u_0 dx > 8\pi/\beta$ only for $n = 2$.

For Cauchy problem: It is also possible for the solution of the problem (1.1) and (1.3) to blow up in finite time when $n \geq 2$ (see [2,14]). The case $n = 2$ is much more interesting in some sense. For the problem of (1.1) and (1.3) with $\kappa = 0$ in $\mathbb{R}^2$, the authors showed that a bounded solution exists globally if the initial mass $\int_{\mathbb{R}^2} u_0 dx < 8\pi/\beta$, whereas the finite time blow-up occurs if the initial mass is larger than $8\pi/\beta$ and $\int_{\mathbb{R}^2} u_0|x - x_0|^2 dx$ is sufficiently small for some fixed point $x_0 \in \mathbb{R}^2$ (see [14]). Other than blow-up results, there are also some conclusions concerning the decay estimates of bounded solution which behaves like the heat kernel, for which we can see in [17,18,23].

The above KS model is devoted to chemoattraction; however, many biological processes also involve chemorepulsion, and can form various interesting biological patterns (see [6,20,26]). In [12], Luca proposed a more general attraction–repulsion chemotaxis model which can be written in the form

$$\begin{cases} \partial_t u - \Delta u = -\beta_1 \nabla \cdot (u \nabla v) + \beta_2 \nabla \cdot (u \nabla w), & x \in \Omega,\ t > 0, \\ \kappa v_t - \Delta v - u + \lambda_1 v = 0, & x \in \Omega,\ t > 0, \\ \kappa w_t - \Delta w - u + \lambda_2 w = 0, & x \in \Omega,\ t > 0, \end{cases} \tag{1.4}$$

where $u(x,t)$ denotes the density of cells, $w(x,t)$ denotes the concentration of chemorepellents, and $v(x,t)$ denotes the concentration of chemoattractants. The parameters β_2 and β_1 represent the sensitivities of cells to the chemorepellents and the chemoattractants respectively; λ_2 and λ_1 are positive parameters and $\kappa = 0, 1$. The case of $\kappa = 0$ can be seen as the approximate version of the case $\kappa = 1$, since the chemicals