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Large-time behavior of an attraction–repulsion chemotaxis system

Ke Lin *, Chunlai Mu, Liangchen Wang

College of Mathematics and Statistics, Chongqing University, Chongqing 401331, PR China

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A B S T R A C T

In this study, we consider an attraction–repulsion chemotaxis system

$$\begin{cases} u_t = \Delta u - \nabla\cdot(\chi u\nabla v) + \nabla\cdot(\xi u\nabla w), & x\in\Omega,\ t>0,\\ v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0,\\ w_t = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t>0 \end{cases}$$

with homogeneous Neumann boundary conditions, where χ, ξ, α, β, γ and δ are positive parameters. Under the critical condition that $\chi\alpha - \xi\gamma = 0$, we prove that the system possesses a unique global solution, which is uniformly bounded in the physical domain $\Omega\subset R^n$ $(n=2,3)$. In addition, we assert that we can find $\epsilon_0>0$ such that for all of the initial data u_0 that satisfy $\int_\Omega u_0 < \epsilon_0$, the solution of the system approaches the steady state $(\bar u_0, \frac{\alpha}{\beta}\bar u_0, \frac{\gamma}{\delta}\bar u_0)$ exponentially as time tends to infinity, where $\bar u_0 := \frac{1}{|\Omega|}\int_\Omega u_0$.

1. Introduction

In this study, we consider the following attraction–repulsion chemotaxis system

$$\begin{cases} u_t = \Delta u - \nabla\cdot(\chi u\nabla v) + \nabla\cdot(\xi u\nabla w), & x\in\Omega,\ t>0,\\ v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0,\\ w_t = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t>0, \end{cases} \tag{1.1}$$

where $u=u(x,t)$ denotes the cell density, $v=v(x,t)$ represents the concentration of an attractive signal, and $w=w(x,t)$ is the concentration of a repulsive signal. The parameters χ, ξ, α, β, γ, and δ are assumed to be positive.

This model was proposed in [21] for describing the quorum effect in a chemotaxis process and in [18] for describing the aggregation of microglia in Alzheimer's disease. This system describes the competition

* Corresponding author.
E-mail address: shuxuelk@126.com (K. Lin).

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between attractive and repulsive signals, which are both produced by the cells themselves. These signals diffuse much faster than cells, so (1.1) reads

$$\begin{cases} u_t = \Delta u - \nabla \cdot (\chi u \nabla v) + \nabla \cdot (\xi u \nabla w), & x \in \Omega,\ t > 0, \\ 0 = \Delta v + \alpha u - \beta v, & x \in \Omega,\ t > 0, \\ 0 = \Delta w + \gamma u - \delta w, & x \in \Omega,\ t > 0. \end{cases} \tag{1.2}$$

If we simplify the chemical processes above by assuming that there is no chemorepulsive signal (i.e., $\xi = 0$), we obtain the well-known (attractive) chemotaxis model, which was proposed by Keller and Segel [13]. Based on an apparent Lyapunov functional for this system of two coupled equations, some basic properties have been discussed in previous studies, such as the global solvability, blow up, and large-time asymptotics (see [5,7–10,19,26] and the references therein). However, if v is decoupled from (1.1), the first and third equations of (1.1) form a repulsive chemotaxis model. In [3], the existence of a Lyapunov functional for this case was proved and the global boundedness of the solutions was derived for $n = 2, 3$ and 4.

Unlike the chemotaxis model with a single chemical, it appears to be difficult to find a Lyapunov functional for (1.1) or (1.2), and thus the mathematical analysis is more challenging. If the repulsion prevails over attraction in the sense that $\chi\alpha - \xi\gamma < 0$, then a unique global classical solution to (1.1) or (1.2) exists in one or more dimensions [17,22,16,11]. However, the solution to (1.2) may blow up in finite time in a bounded domain $\Omega \subset R^2$ if the attraction dominates (i.e., $\chi\alpha - \xi\gamma > 0$) [4,22]. The periodic patterns of (1.1), which are caused by the competitive interaction between attraction and repulsion, were studied in [15] for various parameters.

Most of the results based on (1.1) or (1.2) appear to focus on the question of global existence versus blow up. To the best of our knowledge, few results have been reported that involve the qualitative behavior of (1.1) or (1.2) in higher dimensions. In [12], for any $\beta > 0$ and $\delta > 0$, the large-time behavior of (1.1) was explored in the one-dimensional case. For a higher-dimensional case, [22] showed that each solution of (1.1) or (1.2) converges to a unique trivial stationary solution under the conditions that $\chi\alpha - \xi\gamma < 0$ and $\beta = \delta$.

The aim of the present study is to provide some further insights into the global dynamics of (1.1) in the case where $\chi\alpha - \xi\gamma = 0$. Thus, we impose boundary and initial conditions to close system (1.1). Therefore, we may suppose that

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0, \tag{1.3}$$

and

$$u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), \qquad w(x,0) = w_0(x), \quad x \in \Omega \tag{1.4}$$

satisfy

$$\begin{cases} u_0 \in C^0(\bar{\Omega}) \text{ is nonnegative with } u_0 \not\equiv 0, \quad \text{and} \\ v_0 \in W^{1,\infty}(\Omega) \text{ and } w_0 \in W^{1,\infty}(\Omega) \text{ are nonnegative.} \end{cases} \tag{1.5}$$

Let

$$\chi\alpha - \xi\gamma = 0. \tag{1.6}$$

Then, the model (1.1), (1.3), (1.4) can be changed into the following system