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Boundary concentrating solutions for an anisotropic planar nonlinear Neumann problem with large exponent

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A B S T R A C T

Let Ω be a bounded domain in $\mathbb{R}^2$ with smooth boundary, we study the following anisotropic elliptic problem

$$\begin{cases} -\nabla\big(a(x)\nabla u\big) + a(x)u = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = u^p & \text{on } \partial\Omega, \end{cases}$$

where ν denotes the outer unit normal vector to $\partial\Omega$, $p > 1$ is a large exponent and $a(x)$ is a positive smooth function. We construct solutions of this problem which exhibit the accumulation of arbitrarily many boundary peaks at any isolated local maximum point of $a(x)$ on the boundary.

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1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^2$ with smooth boundary. This paper is concerned with the analysis of solutions to the boundary value problem

$$\begin{cases} -\nabla\big(a(x)\nabla u\big) + a(x)u = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = u^p & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where ν denotes the outer unit normal vector to $\partial\Omega$, $p > 1$ is a large exponent and $a(x)$ is a smooth function over $\overline{\Omega}$ satisfying $(H): 0 < a_1 \le a(x) \le a_2 < +\infty$. Let us define the Rayleigh quotient

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$$I_p(u) = \frac{\int_\Omega a(x)(|\nabla u|^2 + u^2)}{(\int_{\partial\Omega} a(x)|u|^{p+1})^{\frac{2}{p+1}}} \quad \text{for any } u \in H^1(\Omega) \setminus \{0\},$$

and set

$$S_p = \inf_{u \in H^1(\Omega)\setminus\{0\}} I_p(u).$$

From the property of $a(x)$ in (H) and the compactness of the trace Sobolev embedding $H^1(\Omega) \hookrightarrow L^{p+1}(\partial\Omega)$, standard variational method shows that some solutions of (1.1) can be obtained as appropriately scaled extremals of S_p. They are known as least energy solutions of problem (1.1).

Now, if we let $a(x)$ be a constant, then Eq. (1.1) turns to be the following isotropic problem

$$\begin{cases} -\Delta u + u = 0 & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = u^p & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

Recently, Takahashi in [11] studied the asymptotic behavior of least energy solutions of (1.2) as the nonlinear exponent p tends to infinity. It was proved that the least energy solutions remain bounded uniformly with respect to p and develop one peak on the boundary. The location of this blow-up point is associated with a critical point of the Robin function $H(x, x)$ on the boundary, where H is the regular part of the Green function of the corresponding linear Neumann problem. More precisely, the Green function $G(x, y)$ is the solution of the problem

$$\begin{cases} -\Delta_x G(x,y) + G(x,y) = 0, & x \in \Omega, \\ \dfrac{\partial G}{\partial \nu_x}(x,y) = 2\pi\delta_y(x), & x \in \partial\Omega, \end{cases}$$

and $H(x, y)$ is the regular part defined as $H(x, y) = G(x, y) + 2\log|x - y|$. In [2,11] the authors conjectured that the limit problem of (1.2) is

$$\begin{cases} \Delta \upsilon = 0 & \text{in } \mathbb{R}^2_+, \\ \dfrac{\partial \upsilon}{\partial \nu} = e^\upsilon & \text{on } \partial\mathbb{R}^2_+, \\ \displaystyle\int_{\partial\mathbb{R}^2_+} e^\upsilon < +\infty, \end{cases} \tag{1.3}$$

and $\lim_{p\to+\infty} \|u_p\|_{L^\infty(\partial\Omega)} = \sqrt{e}$ holds true at least for least energy solutions u_p of (1.2), where $\mathbb{R}^2_+$ denotes the upper half-plane $\{(x_1, x_2) : x_2 > 0\}$ and ν the outer unit normal vector to $\partial\mathbb{R}^2_+$. It is necessary to point out that the results in [7,8,17] imply that problem (1.3) possesses exactly a two-parameter family of solutions

$$\upsilon_{t,\mu}(x_1, x_2) = \log \frac{2\mu}{(x_1 + t)^2 + (x_2 + \mu)^2}, \tag{1.4}$$

where μ is a positive number and $t \in \mathbb{R}$.

However, problem (1.2) may have solutions with an arbitrarily large number of boundary peaks, as shown by Castro in [2]. Indeed, he proved that given any integer $m \geq 1$, problem (1.2) has at least two families of solutions u_p, each of them satisfying

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