



Global existence and temporal decay for the nematic liquid crystal flows [☆]



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ABSTRACT

This paper is concerned with the Cauchy problem of the three-dimensional nematic liquid crystals. We establish the global well-posedness and time decay rates of the classical solutions with smooth initial data which are of small energy by using Fourier splitting technique and pure energy method.

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1. Introduction

In this paper, we are interested in the following hydrodynamic system modeling the flow of the nematic liquid crystal materials in \mathbb{R}^3 :

$$u_t + u \cdot \nabla u + \nabla \pi = \mu \Delta u - \lambda \nabla d \cdot \Delta d, \quad (1.1)$$

$$d_t + u \cdot \nabla d = \nu (\Delta d + |\nabla d|^2 d), \quad (1.2)$$

$$\operatorname{div} u = 0, \quad (1.3)$$

$$(u, d)(x, 0) = (u_0, d_0)(x), \quad |d_0| = 1. \quad (1.4)$$

Here $u : \mathbb{R}^3 \times [0, \infty) \rightarrow \mathbb{R}^3$ is the velocity field, $\pi : \mathbb{R}^3 \times [0, \infty) \rightarrow \mathbb{R}$ is the pressure, and $d : \mathbb{R}^3 \times [0, \infty) \rightarrow \mathbb{S}^2$ (the unit sphere in \mathbb{R}^3 , i.e. $|d| = 1$) represents the averaged macroscopic/continuum molecular orientations. The positive constants μ , λ and ν represent viscosity, the competition between kinetic energy and potential

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energy, and the microscopic elastic relaxation time for the molecular orientation field, respectively. Without loss of generality, we assume $\mu = \lambda = \nu = 1$ in the sequel.

Liquid crystal is a substance that exhibits a phase of matter that has properties between those of a conventional liquid and those of a solid crystal (cf. [5]). The hydrodynamic flow of incompressible liquid crystals was first derived by Ericksen and Leslie in 1960s (see [6,7,14,15]). Later, Lin [17] proposed the simplified version (1.1)–(1.4), which still retains most of the interesting mathematical properties of the original Ericksen–Leslie model.

Roughly speaking, (1.1)–(1.4) is a nonlinear coupling of the homogeneous Navier–Stokes equations and the system for the flow of harmonic maps into sphere. In the last decades, there were a considerable number of papers devoted to nematic liquid crystal flows. To our knowledge, mathematical analysis of liquid crystals flow was initially studied by a series of papers: Lin [17], and Lin and Liu [19,20], where they investigated an approximation model of the Ericksen–Leslie system by Ginzburg–Landau functionals, that is, Eq. (1.2) is replaced by

$$d_t + u \cdot \nabla d = \nu \left(\Delta d + \frac{1}{\varepsilon^2} (1 - |d|^2) d \right). \quad (1.5)$$

More precisely, the authors proved the global existence of classical and weak solutions of (1.1), (1.3), (1.5) in dimensions two and three, respectively. For any fixed ε , Lin and Liu [20] extended the classical theorem by Caffarelli, Kohn and Nirenberg [1] that asserts the one-dimensional parabolic Hausdorff measure of the singular set of any suitable weak solution is zero. Furthermore, Lin and Liu [21] proved existence of solutions for the general Ericksen–Leslie system and also analyzed the limits of weak solutions of (1.1), (1.3), (1.5) as $\varepsilon \rightarrow 0$.

Compared to (1.5), it is indeed much more difficult to deal with system (1.1)–(1.4) due to the presence of the nonlinear term $|\nabla d|^2 d$ with the restriction $|d| = 1$. In two independent papers [10] and [18], Hong and Lin, Lin and Wang showed the global existence of Leray–Hopf-type weak solutions of (1.1)–(1.4) in dimension two. Since Chang, Ding and Ye [3] show that the strong solutions of the heat flow of harmonic maps must blow up in finite time, one cannot expect that (1.1)–(1.4) has a global smooth solution with general initial data. Huang and Wang [11] announced the local existence of smooth solutions and established a blow-up criterion for this local solutions. Indeed, they gave if the initial velocity $u_0 \in H^m(\mathbb{R}^2)$ and $d_0 \in H^{m+1}(\mathbb{R}^2, \mathbb{S})$ with $\operatorname{div} u_0 = 0$ for $m \geq 3$, then system (1.1)–(1.4) has a unique local classical solution (u, d) on $\mathbb{R}^3 \times [0, T_*)$ satisfying

$$\begin{aligned} u &\in C([0, T_*]; H^m(\mathbb{R}^3)) \cap L^2((0, T_*); H^{m+1}(\mathbb{R}^3)), \\ d &\in C([0, T_*]; H^{m+1}(\mathbb{R}^3, \mathbb{S}^2)) \cap L^2((0, T_*); H^{m+2}(\mathbb{R}^3, \mathbb{S}^2)), \end{aligned}$$

where $0 < T_*$ is constant depending only on the initial value. With small initial data, the global existence of solution of (1.1)–(1.4) was studied in [18,32] and [16,9,29] in two and three dimensions, respectively. Moreover, in order to understand which quantity becomes infinite as the time approaches, various blow-up criteria were established in [8,11,22,23,30,31,33,36,37] and the references therein.

In this paper, we are mainly interested in the global existence and the decay estimates of the smooth solution to (1.1)–(1.4) in H^m ($m \geq 3$) with small energy. We use Fourier splitting method to give the decay estimates. Concerning the time decay rate for the Navier–Stokes equations, besides Fourier splitting method, a new method was established [34] by Zhou and it was applied in [35]. We state the main theorem of the paper as follows.

Theorem 1.1. *Assume the initial data (u_0, d_0) satisfies for an integer $m \geq 3$*

$$u_0 \in H^m(\mathbb{R}^3) \quad \text{with } \operatorname{div} u_0 = 0, \quad d_0 \in H^{m+1}(\mathbb{R}^3, \mathbb{S}^2). \quad (1.6)$$

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