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Orthogonal decomposition and asymptotic behavior for nonlinear Maxwell's equations

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ABSTRACT

We consider nonlinear Maxwell's equations in which the electric conductivity strongly depends on the electric field, acting like a switch-like function for the electromagnetic field. We combine multiplier techniques and Nakao's Method to show that the solution decays at polynomial rates if the conductivity satisfies suitable conditions. Under more restrictive hypotheses, exponential decay is obtained. As a byproduct, we also obtain the decay of the solution of quasi-stationary Maxwell's equations.

1. Introduction

In this work we consider the Maxwell equations (see [10])

$$\epsilon \mathbf{E}_t + \sigma\big(x, |\mathbf{E}|\big)\mathbf{E} - \nabla \times \mathbf{H} = 0, \quad \text{in } \Omega \times (0, +\infty), \tag{1.1}$$

$$\mu \mathbf{H}_t + \nabla \times \mathbf{E} = 0, \quad \text{in } \Omega \times (0, +\infty), \tag{1.2}$$

$$div(\mu \mathbf{H}) = 0, \quad \text{in } \Omega \times (0, +\infty) \tag{1.3}$$

with initial and boundary conditions

$$\mathbf{E}(x,0) = \mathbf{E}_0(x), \qquad \mathbf{H}(x,0) = \mathbf{H}_0(x), \quad \text{in } \Omega, \tag{1.4}$$

$$\eta \times \mathbf{E} = 0 \quad \text{and} \quad \eta \cdot \mathbf{H} = 0 \quad \text{on } \Gamma \times (0, +\infty). \tag{1.5}$$

Here Ω is a bounded, open, simply-connected domain of $\mathbb{R}^3$ with a regular boundary $\Gamma = \partial\Omega$. The vectorial functions $\mathbf{E} = \mathbf{E}(x,t) = (E_1(x,t), E_2(x,t), E_3(x,t))$ and $\mathbf{H} = \mathbf{H}(x,t) = (H_1(x,t), H_2(x,t), H_3(x,t))$ (hereafter, a bold letter means a vector or a vector function in $\mathbb{R}^3$) represent, respectively, the electric field

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and the magnetic field at location $x \in \Omega$ and time t. In (1.5), η is the outward normal on Γ. In (1.1)–(1.2), $\nabla \times \mathbf{v}$ indicates the curl of the vectorial function $\mathbf{v}$ and ϵ and μ are positive constants, characteristics of the medium considered called, respectively, the permittivity and the magnetic permeability. $\sigma = \sigma(x, s)$ is a real valued function representing the electric conductivity, related to Ohm's law.

The mathematical model (1.1)–(1.3) consists of the classical Maxwell's equations in the bounded domain Ω with perfectly conducting boundary $\Gamma = \partial\Omega$. In fact, if $\mathbf{E}(x, t)$ and $\mathbf{H}(x, t)$ denote the electric and magnetic fields in Ω, respectively, and $\mathbf{D}(x, t)$ and $\mathbf{B}(x, t)$ are the electric displacement and magnetic induction in Ω, respectively, then hold (see [10]) the *Faraday law*

$$\nabla \times \mathbf{E} = -\mathbf{B}_t, \tag{1.6}$$

the *Ampere law*

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{D}_t, \tag{1.7}$$

where $\mathbf{J}$ represents the current density, and the *Gauss law for magnetism*

$$div\, \mathbf{B} = 0. \tag{1.8}$$

In our case, we will assume the constitutive relations

$$\mathbf{D} = \epsilon\mathbf{E}, \qquad \mathbf{B} = \mu\mathbf{H} \tag{1.9}$$

and *Omh's law*

$$\mathbf{J} = \sigma\mathbf{E} \tag{1.10}$$

and take, for simplicity, ϵ and μ as positive constants. The boundary condition (1.5) is consistent with the fact that the boundary Γ is perfectly conducting, so that the tangential component of the electric field must vanish.

In some important applications (see, for instance, [1] and [11]) the electric conductivity, σ, depends on the electric field $|\mathbf{E}|$ and may act like a switch-like function (more on this subject can be seen in [15]), hence the usefulness of considering Eq. (1.1).

We shall assume the following hypotheses on the electric conductivity $\sigma(x, s)$.

H(1.1): $\sigma(x, s)$ is measurable in $\Omega \times [0, \infty)$, monotone increasing in s and, for some $0 \le p \le 4$ and for $s \ge 0$,

(i) $$\sigma(x, s)s^2 \ge \sigma_0 s^{p+2},$$

(ii) $$0 \le \sigma(x, s) \le \sigma_1\big(1 + s^p\big),$$

where σ_0, σ_1 are constants, $\sigma_0, \sigma_1 > 0$.

Remark 1.1. $\sigma(x, s)$ satisfies, in particular, the hypotheses **H(2,1)** of [15].

The existence and asymptotic behavior as $t \to \infty$ of solutions of nonlinear Maxwell's equations have been studied by many authors. H.M. Yin [15] considered the singular limit problem for system (1.1)–(1.2) as $\epsilon \to 0$. More precisely, the author showed that, under suitable hypothesis on $\sigma(x, s)$, the system has a unique weak solution and the solution, as $\epsilon \to 0$, converges to the solution of quasi-stationary Maxwell's equations. On the asymptotic behavior of nonlinear Maxwell's system, we can cite important contributions of