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Periodic second order superlinear Hamiltonian systems

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We consider periodic solutions for superlinear second order non-autonomous dynamical systems including both kinetic and potential terms. We study the existence of nontrivial and ground state solutions.

1. Introduction

We consider the following problem. One wishes to solve

$$-\ddot{x}(t) = B(t)x(t) + \nabla_x V\big(t, x(t)\big), \tag{1}$$

where

$$x(t) = \big(x_1(t), \cdots, x_n(t)\big) \tag{2}$$

is a map from $I = [0, T]$ to $\mathbb{R}^n$ such that each component $x_j(t)$ is a periodic function in H^1 with period T, and the function $V(t, x) = V(t, x_1, \cdots, x_n)$ is continuous from $\mathbb{R}^{n+1}$ to $\mathbb{R}$, continuously differentiable with respect to the x_j with

$$\nabla_x V(t, x) = (\partial V/\partial x_1, \cdots, \partial V/\partial x_n) \in C\big(\mathbb{R}^{n+1}, \mathbb{R}^n\big). \tag{3}$$

For each $x \in \mathbb{R}^n$, the function $V(t, x)$ is periodic in t with period T.

We shall study this problem under several sets of assumptions. The elements of the symmetric matrix $B(t)$ are to be real-valued functions $b_{jk}(t) = b_{kj}(t)$. Our assumption on $B(t)$ is:

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$(B1)$ Each component of $B(t)$ is an integrable function on I, i.e., for each j and k, $b_{jk}(t) \in L^1(I)$.

This assumption implies that there is an extension $\mathcal{D}$ of the operator

$$\mathcal{D}_0 x = -\ddot{x}(t) - B(t)x(t)$$

having a discrete, countable spectrum consisting of isolated eigenvalues of finite multiplicity with a finite lower bound $-L$

$$-\infty < -L \le \lambda_0 < \lambda_1 < \lambda_2 < \ldots < \lambda_l < \ldots . \tag{4}$$

Let λ_l be the first positive eigenvalue of $\mathcal{D}$. We allow $\lambda_{l-1} = 0$. Let H be the set of vector functions $x(t)$ described above. It is a Hilbert space with norm satisfying

$$\|x\|_H^2 = \sum_{j=1}^n \|x_j\|_{H^1}^2.$$

We also write

$$\|x\|^2 = \sum_{j=1}^n \|x_j\|^2,$$

where $\|\cdot\|$ is the $L^2(I)$ norm. Define the subspaces M and N of H as,

$$N = \bigoplus_{k<l} E(\lambda_k), \qquad M = N^\perp, \qquad H = M \oplus N,$$

where $E(\lambda_k)$ is the eigenspace of λ_k. Let

$$G(x) = d(x) - 2\int_I V(t,x)\,dt, \tag{5}$$

where $d(x) = (\mathcal{D}x, x)$ (cf. the next section). Let

$$x(t) = w(t) + v(t), \quad w(t) \in M,\ v(t) \in N.$$

We write

$$G_\lambda(x) = \lambda d(w) + d(v) - 2\int_I V(t,x)\,dt, \quad 0 < \lambda < \infty. \tag{6}$$

We let $\mathcal{D}_\lambda$ be the operator corresponding to $d_\lambda(x) = \lambda d(w) + d(v)$. We have

Theorem 1.1. *Assume*

1.
$$2V(t,x) \ge \lambda_{l-1}|x|^2, \quad t \in I,\ x \in \mathbb{R}^n.$$

2.
$$V(t,x)/|x|^2 \to \infty, \quad \textit{as } |x| \to \infty.$$