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Optimal exit strategies for investment projects

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ABSTRACT

We study the problem of an optimal exit strategy for an investment project which is unprofitable and for which the liquidation costs evolve stochastically. The firm has the option to keep the project going while waiting for a buyer, or liquidating the assets at immediate liquidity and termination costs. The liquidity and termination costs are governed by a mean-reverting stochastic process whereas the rate of arrival of buyers is governed by a regime-shifting Markov process. We formulate this problem as a multidimensional optimal stopping time problem with random maturity. We characterize the objective function as the unique viscosity solution of the associated system of variational Hamilton–Jacobi–Bellman inequalities. We derive explicit solutions and numerical examples in the case of power and logarithmic utility functions when the liquidity premium factor follows a mean-reverting CIR process.

1. Introduction

There is often a time when a firm is engaged in a project that does not produce to its full potential and faces the difficult dilemma of shutting it down or keeping it alive in the hope that it will become profitable once again. When an investment in not totally irreversible, assets can be sold at their scrap value minus some liquidation and project termination costs, which may include for example termination pay to workers, legal fees and a liquidity premium in the case of fire sale of the assets. Since these closing costs may be substantial, it may be worthwhile to wait for the project to be profitable again or to wait for an interested buyer that will pay the fair value of the assets and put them to better use. In this study, we give an analytical solution to this

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problem when the liquidation costs and the value of the assets are diffusion processes and the arrival time of a buyer is modeled by means of an intensity function depending on the current state of a Markov chain.

There is a vast literature on firm's investment decisions in stochastic environments, see for instance [2,4,8,15,18,19,23]. In relation to our study, Dixit and Pindyck [9] consider various firm's decisions problems with entry, exit, suspension and/or abandonment scenarios in the case of an asset given by a geometric Brownian motion. The firm's strategy can then be described in terms of stopping times given by the time when the value of the assets hit certain threshold levels characterized as free boundaries of a variational problem. Duckworth and Zervos [10], and Lumley and Zervos [16] solve an optimal investment decision problem with switching costs in which the firm controls the production rate and must decide at which time it exits and re-enters production.

The firm, we consider, in this paper, must decide between liquidating the assets of an underperforming project and waiting for the project to become once again profitable, in a setting where the liquidation costs and the value of the assets are given by general diffusion processes. We formulate this two-dimensional stochastic control problem as an optimal stopping time problem with random maturity and regime shifting.

Amongst the large literature on optimal stopping problems, we may refer to some related works including Bouchard, El Karoui and Touzi [1], Carr [3], Dayanik and Egami [6], Dayanik and Karatzas [7], Guo and Zhang [12], Lamberton and Zervos [13]. In [7] and [13], the authors study optimal stopping problems with general 1-dimensional processes. Random maturity in optimal stopping problem was introduced in [3] and [1]. It allowed to reduce the dimension of their problems as well as addressing the numerical issues. We may refer to Dayanik and Egami [6] for a recent paper on optimal stopping time and random maturity. For optimal stopping problem with regime shifting, we may refer to Guo and Zhang [12], where an explicit optimal stopping rule and the corresponding value function in a closed form are obtained.

In this paper, our optimal stopping problem combines all the above features, i.e., random maturity and regime shifting, in the bi-dimensional framework. We are able to characterize the value function of our problem and provide explicit solution in some particular cases where we manage to reduce the dimension of our control problem.

In the general bi-dimensional framework, the main difficulty is related to the proof of the continuity property and the PDE characterization of the value function. Since it is not possible to get the smooth-fit property, the PDE characterization may be obtained only via the viscosity approach. To prove the comparison principle, one has to overcome the non-linearity of the lower and upper bounds of the value function when building a strict supersolution to our HJB equation.

In the particular cases where it is possible to reduce our problem to a one-dimensional problem, we are able to provide explicit solution. Our reduced one-dimensional problem is highly related to previous studies in the literature, see for instance Zervos, Johnson and Alazemi [24] and Leung, Li and Wang [14].

The rest of the paper is organized as follows. We define the model and formulate our optimal stopping problem in the following section. In Section 3, we characterize the solution of the problem in terms of the unique viscosity solution to the associated HJB system and obtain some qualitative description of these functions. In Sections 4 and 5, we derive explicit solutions in the case of power and logarithmic utility functions when the liquidity discount factor follows a mean-reverting CIR process, and provide numerical examples.

2. The investment project

Let $(\Omega, \mathbb{F}, \mathbb{P})$ be a probability space equipped with a filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$, satisfying the usual conditions. It is assumed that all random variables and stochastic processes are defined on the stochastic basis $(\Omega, \mathbb{F}, \mathbb{P})$. We denote by $\mathcal{T}$ the collection of all $\mathbb{F}$-stopping times. Let W and B be two correlated $\mathbb{F}$-Brownian motions, with correlation ρ, i.e. $d[W, B]_t = \rho dt$ for all t.

We consider a firm which owns assets that are currently locked up in an investment project which currently produces no output per unit of time. Because the firm is currently not using the assets at its full potential,