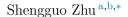
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# Blow-up criterion for the compressible magnetohydrodynamic equations with vacuum



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#### ABSTRACT

In this paper, the 3-D compressible magnetohydrodynamic (MHD) equations with initial vacuum or infinite electric conductivity is considered. We prove that the  $L^{\infty}$  norms of the deformation tensor D(u) and the absolute temperature  $\theta$  control the possible blow-up (see [18,23]) of strong solutions, especially for the non-resistive MHD system when the magnetic diffusion vanishes. This conclusion means that if a solution of the compressible MHD equations is initially regular and loses its regularity at some later time, then the formation of singularity must be caused by losing the bound of D(u) or  $\theta$  as the critical time approaches. The viscosity coefficients are only restricted by the physical conditions. Our criterion (see (1.17)) is similar to [17] for 3-D incompressible Euler equations and to [12] for 3-D compressible isentropic Navier–Stokes equations.

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### 1. Introduction

Magnetohydrodynamics is that part of the mechanics of continuous media which studies the motion of electrically conducting media in the presence of a magnetic field. The dynamic motion of fluid and magnetic field interact strongly with each other, so the hydrodynamic and electrodynamic effects are coupled. The applications of magnetohydrodynamics cover a very wide range of physical objects, from liquid metals to cosmic plasmas, for example, the intensely heated and ionized fluids in an electromagnetic field in astrophysics and plasma physics. In 3-D space, the compressible magnetohydrodynamic equations in a domain  $\Omega$  of  $\mathbb{R}^3$  can be written as



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$$\begin{cases} H_t - \operatorname{rot}(u \times H) = -\operatorname{rot}\left(\frac{1}{\sigma} \operatorname{rot} H\right), \\ \operatorname{div} H = 0, \\ \rho_t + \operatorname{div}(\rho u) = 0, \\ (\rho u)_t + \operatorname{div}(\rho u \otimes u) + \nabla P = \operatorname{div} \mathbb{T} + \operatorname{rot} H \times H, \\ (\rho \theta)_t + \operatorname{div}(\rho \theta u) - \kappa \Delta \theta + P \operatorname{div} u = \operatorname{div}(u\mathbb{T}) - u \operatorname{div} \mathbb{T} + \frac{1}{\sigma} |\operatorname{rot} H|^2. \end{cases}$$
(1.1)

In this system,  $x \in \Omega$  is the spatial coordinate;  $t \ge 0$  is the time;  $H = (H^1, H^2, H^3)$  is the magnetic field; rot  $H = \nabla \times H$  denotes the rotation of the magnetic field;  $0 < \sigma \le \infty$  is the electric conductivity coefficient;  $\rho$  is the mass density;  $u = (u^1, u^2, u^3) \in \mathbb{R}^3$  is the velocity of fluids;  $\kappa > 0$  is the thermal conductivity coefficient; P is the pressure satisfying

$$P = R\rho\theta, \tag{1.2}$$

where  $\theta$  is the absolute temperature, R is a positive constant; T is the viscosity stress tensor:

$$\mathbb{T} = 2\mu D(u) + \lambda \operatorname{div} u \mathbb{I}_3, \quad D(u) = \frac{\nabla u + (\nabla u)^\top}{2}, \tag{1.3}$$

where D(u) is the deformation tensor,  $\mathbb{I}_3$  is the  $3 \times 3$  unit matrix,  $\mu$  is the shear viscosity coefficient,  $\lambda + \frac{2}{3}\mu$  is the bulk viscosity coefficient,  $\mu$  and  $\lambda$  are both real constants satisfying

$$\mu > 0, \qquad \lambda + \frac{2}{3}\mu \ge 0, \tag{1.4}$$

which ensures the ellipticity of the Lamé operator. Although the electric field E doesn't appear in system (1.1), it is indeed induced according to a relation

$$E = \frac{1}{\sigma} \operatorname{rot} H - u \times H$$

by moving the conductive flow in the magnetic field.

The aim of this paper is to give a blow-up criterion of strong solutions to system (1.1) in a bounded, smooth domain  $\Omega \in \mathbb{R}^3$  with the initial condition:

$$(H, \rho, u, \theta)|_{t=0} = (H_0(x), \rho_0(x), u_0(x), \theta_0(x)), \quad x \in \Omega,$$
(1.5)

and the Dirichlet, Neumann boundary conditions for  $(H, u, \theta)$ :

$$(H, u, \partial \theta / \partial n)|_{\partial \Omega} = (0, 0, 0), \quad \text{when } 0 < \sigma < +\infty;$$

$$(1.6)$$

$$(u, \partial \theta / \partial n)|_{\partial \Omega} = (0, 0), \quad \text{when } \sigma = +\infty,$$
 (1.7)

where n is the unit outer normal vector to  $\partial \Omega$ . Actually, some similar result for  $\Omega = \mathbb{R}^3$  can be also obtained via the similar argument used in this paper.

Throughout this paper, we adopt the following simplified notations for the standard homogeneous and inhomogeneous Sobolev space:

$$\begin{split} D^{k,r} &= \left\{ f \in L^1_{loc}(\varOmega) : |f|_{D^{k,r}} = \left| \nabla^k f \right|_{L^r} < +\infty \right\}, \qquad D^k = D^{k,2}, \\ \left\| (f,g) \right\|_X &= \|f\|_X + \|g\|_X, \qquad \|f\|_{1,0} = \|f\|_{H^1_0(\varOmega)}, \qquad \|f\|_s = \|f\|_{H^s(\varOmega)}, \\ \|f\|_{W^{k,r}} &= \|f\|_{W^{k,r}(\varOmega)}, \qquad |f|_p = \|f\|_{L^p(\varOmega)}, \qquad |f|_{D^{k,r}} = \|f\|_{D^{k,r}(\varOmega)}. \end{split}$$

A detailed study of homogeneous Sobolev space may be found in [9].

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