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The stochastic fractional power dissipative equations in any dimension and applications

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ABSTRACT

This paper is concerned with stochastic fractional power dissipative equations with multiplicative noise in n-dimension ($n \geq 1$) space. The well-posedness for the subcritical nonlinearities is proved in appropriate space–time space by the contraction mapping principle and Strichartz estimates. The main result can be applied to various types of SPDEs such as stochastic reaction–diffusion equations, stochastic fractional Burgers equation and stochastic fractional Navier–Stokes equation.

1. Introduction

In the paper, we focus on the following fractional power dissipative equations with multiplicative noise,

$$\begin{cases} \partial_t u + (-\triangle)^\alpha u = Q(D)f(u) + g(t,x,u)\dot{W}(t,x), & (t,x) \in \mathbb{R}^+ \times \mathbb{R}^n, \\ u(0,x) = \varphi(x), & x \in \mathbb{R}^n, \end{cases} \tag{1.1}$$

where $Q(D)$ is a homogeneous pseudo-differential operator of order $d \in [0, 2\alpha)$ with the real number $\alpha > 0$. The process $\dot{W}(t,x)$ is a Gaussian noise, white in time and correlated in space. We will give the assumptions on f, g and W in Section 2.

For the even order (larger than 2) stochastic partial differential equations driven by cylindrical Brownian motion with uniformly bounded Lipschitz drift coefficients and uniformly bounded constant diffusion coefficients were initially studied by T. Funaki [9,10] and then were restudied in [4]. In [8], Debbi and Dozzi

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studied the following Cauchy problem of the stochastic fractional partial differential equation perturbed by space–time white noise

$$\begin{cases} \dfrac{\partial u}{\partial t} = D^{\alpha}_{\delta} u + g(t,x,u) + \displaystyle\sum_{k=1}^{m} \dfrac{\partial^k h_k}{\partial x^k}(t,x,u) + f(t,x,u)\dot{W}(t,x), \\ u(0,x) = \varphi(x), \quad (t,x) \in \mathbb{R}^+ \times \mathbb{R}, \end{cases} \tag{1.2}$$

where $m \in \mathbb{N}$, $1 \le m \le [\alpha]$, $[\alpha]$ is the integer part of α and D^{α}_{δ} is the fractional differential operator with respect to the spatial variable defined by

$$D^{\alpha}_{\delta}\phi(x) = \mathcal{F}^{-1}\big\{-|\cdot|^{\alpha} e^{-i\delta\frac{\pi}{2}\operatorname{sgn}(\cdot)}\mathcal{F}(\phi)\big\}(x),$$

where $\alpha \in (0,\infty)$, $\delta \le \min\{\alpha - [\alpha], 2 + [\alpha]_2 - \alpha\}$, $[\alpha]$ and $[\alpha]_2$ denote the largest integer and the largest even integer no greater than α. It is a generalization of various well-known operators, such as the Laplacian operator (when $\alpha = 2$), the inverse of the generalized Riesz–Feller potential (when $\alpha > 2$) and the Riemann–Liouville operator (when $\delta = \alpha - [\alpha]$, or $2 + [\alpha]_2 - \alpha$). Under Lipschitz and growth conditions, generalizing the method of the stochastic heat equation in the framework of J.B. Walsh [14], Debbi and Dozzi in [8] obtained the existence, uniqueness and regularity of the trajectories of mild solutions of (1.2) of order $\alpha \in (1,+\infty)/\mathbb{N}$ in $L^p(\Omega; C(\mathbb{R}^+ \times \mathbb{R}))$ with $p \ge 2$. Replacing $f(t,x,u)$ in (1.2) by $\sum_{k=1}^{n} \frac{\partial^k f_k}{\partial x^k}(t,x,u)$, the well-posedness of (1.2) of order $\alpha > 1$ in $L^p(\Omega; C(\mathbb{R}^+ \times \mathbb{R}))$ with $p \ge 2$ was obtained in [12], and it improved the Hölder continuous results in [8]. With $h_k = 0$ in (1.2), Boulanba, Eddahbi and Mellouk [1] obtained the well-posedness of (1.2) with spatially correlated noise in n-dimension ($n \ge 1$) space in $L^p(\Omega; C(\mathbb{R}^+ \times \mathbb{R}))$ with $2 \le p \le p_0$, where $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_d)$, $\delta = (\delta_1, \delta_2, \dots, \delta_d)$ and $\alpha_i \in (0,2]/\{1\}$, $|\delta_i| \le \min\{\alpha_i, 2 - \alpha_i\}$.

In this paper, we consider the well-posedness of (1.2) in the n-dimension space with $\delta = 0$ and $\alpha_i = \alpha_j$, $i \ne j$, i.e. Eq. (1.1). This model includes a large number of nonlinear models in fluid dynamics such as stochastic reaction–diffusion equations, stochastic fractional Burgers equation and stochastic Navier–Stokes equation. Motivated by [7], we give a unified method to deal with the existence and uniqueness of solutions for the Cauchy problem (1.1) with initial data in $L^{\rho}(\Omega; L^2(\mathbb{R}^n))$, employing appropriate space–time spaces such as $L^{\rho}(\Omega; \mathcal{E})$, $\mathcal{E} = C([0,T]; L^2(\mathbb{R}^n)) \cap L^r([0,T]; L^p(\mathbb{R}^n))$. We will use the contraction mapping principle to get the well-posedness of (1.1). Then, the global existence will be obtained if there are some energy estimates. Relative results in [1] will be generalized to all fractional order $\alpha > 0$ in this paper. The well-posedness of solutions for stochastic fractional Burgers equation in [2] can also be improved in some sense in our paper (see Example 4.2).

This paper is organized as follows: In Section 2, some necessary notations and preliminary estimates are given. In Section 3, the main result and the corresponding proof are given. In the last Section, some examples are give to illustrate our result.

2. Preliminary estimates

In the present section, the space–time estimates for the linear equation and stochastic integral in those space will be given. Before stating our results precisely, we introduce some notations and provide some useful lemmas.

Given two separable Hilbert Spaces H and $\widetilde{H}$, we denote by $L^2_0(H;\widetilde{H})$ the space of Hilbert–Schmidt operators from H into $\widetilde{H}$. The corresponding norm is then given by $\|\Phi\|^2_{L^2_0(H;\widetilde{H})} = tr(\Phi^*\Phi) = \sum_{k\in\mathbb{N}} |\Phi e_k|^2_{\widetilde{H}}$, where $(e_k)_{k\in\mathbb{N}}$ is any orthonormal basis of H. When $H = \widetilde{H} = L^2(\mathbb{R}^n)$, $L^2_0(H;\widetilde{H})$ is simply denoted by L^0_2.

Given a Banach space B, we will also consider a γ-radonifying operator [3] $K : H \to B$ such that the image by K of the canonical Gaussian distribution on H extends to a Borel probability measure on B. We denote it as $R(H;B)$. As shown by Proposition 7.1 in [3] and Lemma 5.6 in [13], this is equivalent to the