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Analysis of a set of nonlinear dynamics trajectories: Stability of difference equations ☆

A.Yu. Aleksandrov [a], A.A. Martynyuk [b,*], A.V. Platonov [a]

[a] Saint Petersburg State University, 35 Universitetskij Pr., Petrodvorets, St. Petersburg, 198504, Russia
[b] S.P. Timoshenko Institute of Mechanics of NAS of Ukraine, 3 Nesterova Str., Kiev, 03057, Ukraine

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ABSTRACT

For a set of difference equations generated by discretization of the set of differential equations with Hukuhara derivative a principle of comparison with matrix Lyapunov function is specified and sufficient stability conditions of certain type are established. The analysis is carried out in terms of a matrix Lyapunov function of special structure. For an essentially nonlinear multiconnected switched difference system, conditions are obtained providing the asymptotic stability of its zero solution for any switching law. An example is presented to demonstrate efficiency of the proposed approaches.

1. Introduction

Difference equations are widely applied in probability theory, queueing theory, theory of stochastic power series, number theory, power systems theory, economics, ecology and in the analysis of other real world phenomena [1,8,9]. While the general stability theory of classical difference equations is well-developed, see [1,4,8,9,12,16,19,24] and references cited therein, the theory of a set of difference equations is in a primitive state.

In particular, in [14] and [5] an extension of some results obtained for the set of continuous systems with Hukuhara derivative was proposed for a set of difference equations. Unsolved problem is the problem of constructing an appropriate Lyapunov function satisfying special properties needed for zero solution stability to be established.

An important problem requiring the study of a set of difference equation systems is that of stability analysis of solutions of switched difference systems [6,7,10,11,22]. A switched system consists of a family of

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* Corresponding author. Fax: +380 44 4560319.
E-mail addresses: alex43102006@yandex.ru (A.Yu. Aleksandrov), amartynyuk@voliacable.com (A.A. Martynyuk), al-platon1@yandex.ru (A.V. Platonov).

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subsystems and a switching law determining at each time instant which subsystem is active. Such systems appear in modeling of mechanical systems, electric power systems, technological processes and intelligent control systems with logic-based controllers, etc. [7,17,18,23].

In some cases, it is necessary to design a control system in such a way that it remains stable for any admissible switching law [17,18]. These cases are natural, when switching signal is either unknown or too complicated to be explicitly taken into account. A general approach to the above problem is based on the computation of a common Lyapunov function for a family of subsystems corresponding to the switched system. This approach has been effectively used in many papers (see, for instance, [7,10,17,18,22,23] and references cited therein). However, the problem of existence of a common Lyapunov function is not completely solved even for the case of a family of linear time-invariant systems [18]. This problem is especially difficult for switched systems of high dimension (for large-scale or complex systems).

In the present paper we set out a general approach to stability analysis problem for a set of trajectories of difference equations with uncertain parameter values. After a "regularization" procedure is applied to the initial difference system, a theorem of the comparison method and a generalized Lyapunov function are employed to establish sufficient stability conditions for zero solution. Since the proposed function is constructed in terms of auxiliary (2×2)-matrix function, this allows to weaken the requirements to the dynamic properties of subsystems, in terms of which the matrix function elements are found.

As an application of the general approach, a nonlinear multiconnected (complex) switched difference system is studied. By means of the comparison method, the conditions are obtained under which the zero solution of the system is asymptotically stable for any switching law.

2. Preliminaries

Further we shall need the following notions and results, see [5] and references cited therein. Let $K_C(\mathbb{R}^q)$ denote a family of all nonempty compact and convex subsets in the space $\mathbb{R}^q$; $K(\mathbb{R}^q)$ contain all nonempty compact subsets in $\mathbb{R}^q$, and $C(\mathbb{R}^q)$ be a subset of all nonempty closed subsets in $\mathbb{R}^q$. The distance between nonempty closed subsets A and B of the space $\mathbb{R}^q$ is specified by the formula

$$D[A, B] = \max\{d_H(A, B),\ d_H(B, A)\},$$

where $d_H(B, A) = \sup\{d(b, A) : \ b \in B\}$ is a Hausdorff separation of the sets A and B, and $d(b, A) = \inf\{\|b - a\| : a \in A\}$ is a distance from the point b to the set A, $\|\cdot\|$ is the Euclidean norm.

The pair $(C(\mathbb{R}^q), D)$ is a complete separable metric space, where $K(\mathbb{R}^q)$ and $K_C(\mathbb{R}^q)$ are closed subsets.

Let F be a mapping of the domain Q of the space $\mathbb{R}^q$ into the metric space $(K_C(\mathbb{R}^q), D)$, i.e., $F : Q \to K_C(\mathbb{R}^q)$, which is equivalent to the inclusion $F(t) \in K_C(\mathbb{R}^q)$ for all $t \in Q$. Such mappings are called the multivalued mappings of Q into $\mathbb{R}^q$.

Let $\mathbb{N}$ denote a set of positive integers, $\mathbb{N}_+ = \mathbb{N} \cup \{0\}$, and we designate by $\mathbb{N}_{n_0}$ the set

$$\mathbb{N}_{n_0} = \{n_0, n_0 + 1, \dots, n_0 + k, \dots\},$$

where $k \in \mathbb{N}$ and $n_0 \in \mathbb{N}_+$.

We recall a result from the theory of classical difference equations, which will be of further need. It is worth noting that, in the statement of this result, vector inequalities are understood in the component-wise sense: if $a, b \in \mathbb{R}^q$, then the inequality $a \geq b$ means that $a_i \geq b_i$ for all $i = 1, \dots, q$.

Theorem 1. *(Cf. [15].) Let the function $G(n, U)$, $G : \mathbb{N}_+ \times \mathbb{R}^q_+ \to \mathbb{R}^q$, satisfy the condition*

$$G(n, \tilde{U}) \leq G(n, \hat{U})$$