J. Math. Anal. Appl. 421 (2015) 260–275

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Limit cycle bifurcations by perturbing piecewise smooth Hamiltonian systems with multiple parameters ☆

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ARTICLE INFO

Article history:
Received 8 March 2014
Available online 11 July 2014
Submitted by J. Shi

Keywords:
Limit cycle
Periodic orbit
Melnikov function
Piecewise smooth system

ABSTRACT

This paper is concerned with the problem of limit cycle bifurcation for piecewise smooth near-Hamiltonian systems with multiple parameters. By the first Melnikov function, some novel criteria have been established for the existence of multiple limit cycles. Furthermore, an example is included to validate the obtained results by considering the maximum number of limit cycles for a piecewise quadratic system studied in Llibre and Mereu (2014) [12]. Compared with the result in the above reference, one more limit cycle is found by our method.

1. Introduction and main results

Piecewise smooth systems were widely investigated in the past few decades [1,2,5,6,9–14,16,20]. Until now, numerous results have been developed, one of which is to determine the number of limit cycles and their relative distributions for this kind of systems, such as in [5,9–14,16,20] and references therein. In particular, limit cycles of piecewise linear systems defined on two half-planes separated by a straight line $x = 0$ or $y = 0$ have received considerable attention recently in the literature [9,14,16]. It is found that piecewise linear systems can have three limit cycles [9,14]. While, to the best of our knowledge, there are few studies on the maximal number of limit cycles for piecewise quadratic systems. The main difficult lies in that one does not know how to determine the maximal number of limit cycles for quadratic systems. So far, it has been shown that there exist four limit cycles for such systems [3,19]. Up to now, there is no better results to cover it.

As an attempt to make further investigation for the piecewise quadratic system, it is also good to consider a perturbation system of a quadratic polynomial differential system with an isochronous center and determine the maximum number of the limit cycles for it. The author [17] provided a reference for the classification of quadratic polynomial differential systems containing an isochronous center. When the

☆ The project was supported by National Natural Science Foundation of China (11271261) and FP7-PEOPLE-2012-IRSES-316338.
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http://dx.doi.org/10.1016/j.jmaa.2014.07.013

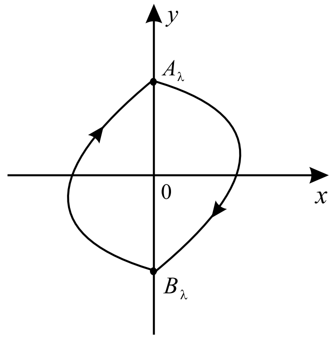

Fig. 1.1. Periodic orbit L_λ.

systems are perturbed by quadratic polynomials, one can find two limit cycles which emerge from the periodic orbits [4]. While when they are perturbed by piecewise quadratic polynomials, one can obtain five limit cycles [12]. This conclusion is improved in this paper, one more limit cycle obtaining.

Similar to the investigation of the limit cycle problem for smooth systems [7,8,18,21], it is well known that there exist at least two methods to study the problem of limit cycles emerging from period annulus for piecewise smooth systems. One is the averaging method established in [13] and another is the Melnikov function method developed in [11]. In this paper, we will utilize the latter one to consider the problem of limit cycles for a class of piecewise smooth near-Hamiltonian systems with multiple parameters by following the main idea of [8]. More precisely, we consider the following system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{cases} \begin{pmatrix} H_y^+(x,y,\lambda) + \varepsilon p^+(x,y,\lambda) \\ -H_x^+(x,y,\lambda) + \varepsilon q^+(x,y,\lambda) \end{pmatrix}, & x > 0, \\ \begin{pmatrix} H_y^-(x,y,\lambda) + \varepsilon p^-(x,y,\lambda) \\ -H_x^-(x,y,\lambda) + \varepsilon q^-(x,y,\lambda) \end{pmatrix}, & x < 0, \end{cases} \tag{1.1}$$

where $0 < \varepsilon \ll \lambda \ll 1$, $H^\pm$, $p^\pm$ and $q^\pm$ are C^∞ functions in (x, y). Without loss of generality, for system (1.1), we make two assumptions below:

(H1) For small λ, suppose $(1.1)|_{\varepsilon=0}$ has a family of periodic orbits with clockwise orientation given by

$$L_\lambda(h): \quad H(x,y,\lambda) = h, \quad h \in I_\lambda.$$

(H2) Each periodic orbit $L_\lambda(h)$ defined in (H1) intersects the y-axis with two different points in turn, denoted by $A_\lambda(h) = (0, a(h,\lambda))$ and $B_\lambda(h) = (0, b(h,\lambda))$ with $a(h,\lambda) > b(h,\lambda)$, respectively, see Fig. 1.1.

Let (H1) and (H2) hold. Then, by Theorem 1.1 in [11] and Lemma 2.2 in [10], the first order Melnikov function of (1.1) can be expressed as

$$M(h,\lambda) = \int_{\widehat{A_\lambda B_\lambda}} q^+ dx - p^+ dy + \frac{H_y^+(A_\lambda,\lambda)}{H_y^-(A_\lambda,\lambda)} \int_{\widehat{B_\lambda A_\lambda}} q^- dx - p^- dy, \tag{1.2}$$

where $\widehat{A_\lambda B_\lambda}$ and $\widehat{B_\lambda A_\lambda}$ denote $L_\lambda \cap \{x \geq 0\}$ and $L_\lambda \cap \{x \leq 0\}$ respectively. We rewrite (1.2) for $\lambda > 0$ small

$$M(h,\lambda) = M_0(h) + \lambda M_1(h) + \lambda^2 M_2(h) + O(\lambda^3). \tag{1.3}$$