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Regularity criterion for generalized Newtonian fluids in bounded domains ☆

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ABSTRACT

The regularity of a non-Newtonian flow in a domain with a boundary cannot be treated easily because of the nonlinear viscosity term and the global property of the pressure. A distance function was used in a previous study to obtain a strong solution. In the present study, using the distance function idea, we obtain Serrin-type regularity criteria for the vorticity and the velocity with regard to non-Newtonian equations with shear-dependent viscosity in a bounded domain.

1. Introduction

We consider unsteady incompressible fluids in a smooth bounded domain $\Omega \subset \mathbb{R}^n$, $n = 2, 3$, which are described by the system

$$\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla \cdot \sigma + \nabla p = \mathbf{0}, \qquad \nabla \cdot \mathbf{u} = 0, \tag{1.1}$$

where $u_t = \frac{\partial u}{\partial t}$, $\nabla = (\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_n})$. The initial condition is given by

$$\mathbf{u}|_{t=0} = \mathbf{a} \tag{1.2}$$

and the Dirichlet boundary condition is given by

$$\mathbf{u}|_{\partial\Omega} = 0, \tag{1.3}$$

where $\mathbf{u} = (u_1, \cdots, u_n)$ denotes the velocity, p is the pressure, σ is the extra stress tensor, and $\mathbf{a}$ is the given initial velocity of the fluid. We consider constitutive relations for σ of the form $\sigma = \sigma(D)$, where D denotes the symmetric part of the velocity gradient and $|D|$ denotes the usual Euclidean matrix-norm, i.e.,

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$D = (D_{ij})_{i,j=1,\cdots,n}$ and $|D|^2 = \sum_{i,j=1}^n D_{ij}^2$ for $D_{ij} = D_{ij}(\mathbf{u}) = \frac{1}{2}(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j})_{i,j=1,\cdots,n}$. In this study, we consider the stress tensor of the form

$$\sigma = \left(1 + |D|^2\right)^{\frac{r}{2}} D, \tag{1.4}$$

where $0 < r$ is a power term that represents the viscosity properties of fluids.

The existence of weak solutions of the system (1.1) with the Dirichlet boundary condition for $r \geq \frac{n-2}{n+2}$ was first reported in [19–21], and the solution is unique for $r > \frac{n-2}{2}$. The existence of measure-valued solutions of system (1.1) with Dirichlet boundary conditions was demonstrated in [22,25]. The existence of a weak solution with the Dirichlet boundary condition was extended to $1 > r \geq 0$ in [24] and the solution is strong for $1 > r \geq \frac{1}{4}$, $n = 3$ (see also [8,27]). In [9], boundary regularity results were provided for $r > \frac{n-2}{n+2}$. The existence result with the Dirichlet boundary condition was extended to the case where $r > -\frac{2}{n+2}$ in [30] and to $r > -\frac{4}{n+2}$ in [15].

Based on these studies, we are particularly interested in the regularity properties of weak solutions. The existence of a local-in-time strong solution with the space periodic boundary condition was shown for $r > -\frac{1}{3}$ in [23], for $0 > r > -\frac{3}{5}$ in [14], and for $0 > r > -1$ in [10]. In particular, the stress tensor was of the form $\sigma = |D|^r D$ in [10]. In [11], the existence of a local-in-time strong solution was shown for any $r \geq -1$ with no-slip or slip boundary conditions (or global time existence for small data). Recently, in [5], regularities were obtained for the model $\sigma = |D|^r D$ instead of (1.4), which are global in time for $r \geq \frac{n-2}{n+2}$ and local in time for $0 > r > \frac{n-4}{n+2}$.

In summary, the following regularity results are known.

Dirichlet boundary condition:

global regularity with $\sigma = |D|^r D$: $r \geq \frac{1}{5}$

short time regularity with (1.4): $r \geq -1$

$$\text{with } \sigma = |D|^r D\text{: } 0 \geq r > -\frac{1}{5}$$

periodic boundary condition:

global regularity with $\sigma = |D|^r D$: $r \geq \frac{1}{5}$

short time regularity with $\sigma = |D|^r D$: $0 \geq r > -1$.

For the slip boundary condition, refer to [8]. The stress tensor σ has various other forms, for example, $(1 + |D|)^r D$ and $(1 + |D|^r)D$. In mathematical analysis, the difficulty of dealing with these tensors might be similar to that of (1.4), with the exception of $|D|^r D$. For more general forms of σ, refer to [23].

In the present study, we determine the Serrin-type regularity criteria for the vorticity in Section 2, and for the velocity in Section 3.

The Serrin regularity criteria for the Navier–Stokes equations ($r = 0$) have been studied by many researchers [16,28,29]. Recently, the two-component regularity criterion for the vorticity was studied in [12]. The two-component regularity criterion for the velocity was introduced in [2] in 1997, which was published in [3], and the one-component regularity criterion was reported in [26]. In addition, many similar results have been reported for related areas of interest [7,13,18]. For non-Newtonian fluids with the periodic boundary condition, Serrin-type regularity criteria were studied in [4] for shear thinning fluids, and in [6] for shear thickening cases.

Remark 1.1. In the present study, we consider the no-slip boundary condition but our results also apply to more general boundary conditions, such as the slip boundary condition, if the short time regularity is guaranteed because we use weight functions that vanish on the boundary for our analysis.