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Some remarks on the asymptotic profiles of solutions for strongly damped wave equations on the 1-D half space

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ABSTRACT

We consider a mixed problem with the Dirichlet null boundary conditions on the 1-dimensional half space $(0, +\infty)$. We will derive an asymptotic profiles of the corresponding solutions in the case when the initial data belong to a weighted $L^{1,2}(0, \infty)$ space by employing a method recently introduced in [7] or [8].

1. Introduction

We are concerned with the Cauchy problem for strongly damped wave equations in $(0, +\infty)$:

$$u_{tt}(t,x) - u_{xx}(t,x) - u_{xxt}(t,x) = 0, \quad (t,x) \in (0,\infty) \times (0,+\infty), \tag{1.1}$$

$$u(0,x) = u_0(x), \qquad u_t(0,x) = u_1(x), \quad x \in (0,+\infty), \tag{1.2}$$

$$u(t,0) = 0, \quad t \in (0,+\infty), \tag{1.3}$$

where the initial data u_0 and u_1 can be chosen such as

$$[u_0, u_1] \in H_0^1(0,\infty) \times L^2(0,\infty).$$

The unique existence of weak solution $u \in C([0,+\infty); H_0^1(0,\infty)) \cap C^1([0,+\infty); L^2(0,\infty))$ to (1.1)–(1.3) can be discussed by the method due to [10]. For the solution $u(t,x)$ and the initial data (u_0, u_1) we can consider the odd extension of $u(t,x)$ and $u_j(x)$ $(j = 0,1)$ to the whole space $(-\infty, \infty)$ as follows:

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$$v(t,x) = \begin{cases} u(t,x) & x \geq 0 \\ -u(t,-x) & x < 0 \end{cases}$$

and for $j = 0, 1$,

$$v_j(x) = \begin{cases} u_j(x) & x \geq 0 \\ -u_j(-x) & x < 0. \end{cases}$$

It is easy to check that the function $v(t,x)$ becomes a solution to the Cauchy problem below:

$$v_{tt}(t,x) - v_{xx}(t,x) - v_{xxt}(t,x) = 0, \quad (t,x) \in (0,\infty) \times \mathbf{R}, \tag{1.4}$$

$$v(0,x) = v_0(x), \qquad v_t(0,x) = v_1(x), \quad x \in \mathbf{R}. \tag{1.5}$$

It should be noticed that the initial data $v_j(x)$ $(j = 0, 1)$ and the solution $v(t,x)$ satisfy

$$\int_{\mathbf{R}} v_j(x)dx = 0 \quad (j = 0, 1), \tag{1.6}$$

and

$$\int_{\mathbf{R}} v(t,x)dx = 0, \quad t > 0. \tag{1.7}$$

By the way, recently Ikehata proved the following result in [8]. In fact, he has dealt with the more general dimensional case.

Theorem 1.1. (*See* [8].) *If* $[v_0, v_1] \in (H^1(\mathbf{R}) \cap L^{1,1}(\mathbf{R})) \times (L^2(\mathbf{R}) \cap L^{1,1}(\mathbf{R}))$*, then the solution* $v(t,x)$ *to problem* (1.4)–(1.5) *satisfies*

$$\int_{\mathbf{R}} \left| \mathcal{F}\big(v(t,\cdot)\big)(\xi) - \left\{ P_1 e^{-t|\xi|^2/2} \frac{\sin(t|\xi|)}{|\xi|} + P_0 e^{-t|\xi|^2/2} \cos\big(t|\xi|\big) \right\} \right|^2 d\xi$$
$$\leq C\big(\|v_1\|_1^2 + \|v_0\|_1^2\big)t^{-\frac{1}{2}} + C\|v_1\|_{1,1}^2 t^{-\frac{1}{2}} + C\|v_0\|_{1,1}^2 t^{-\frac{3}{2}} + Ce^{-\alpha t}\big(\|v_1\|^2 + \|v_0\|^2\big)$$

with some constants $\alpha > 0$ *and* $C > 0$*, where*

$$P_0 := \int_{\mathbf{R}^n} v_0(x)dx, \qquad P_1 := \int_{\mathbf{R}^n} v_1(x)dx.$$

Unfortunately, in our case we have an unlucky property shown by (1.6), that is to say, if one applies Theorem 1.1 to problem (1.4)–(1.5) one can have nothing about the asymptotic profiles of the solution $v(t,x)$ because of (1.6) $(P_0 = P_1 = 0)$. This implies that we can know nothing about the asymptotic profile of the original solution $u(t,x)$, which satisfies $u(t,x) = v(t,x)|_{x\geq 0}$. In connection with this, recently Said-Houari [13] studied asymptotic profiles of solutions for the Cauchy problem of the usual damped wave equation:

$$u_{tt} - \Delta u + u_t = 0, \quad t > 0, \ x \in \mathbf{R}^n,$$

together with initial data $[u_0, u_1]$ satisfying $\int_{\mathbf{R}^n} u_j(x)dx = 0$ $(j = 0, 1)$ plus weight conditions.

The purpose of this paper is to remake Theorem 1.1 in order to catch asymptotic profiles of solutions to problem (1.4)–(1.5) with (1.6), and is to apply the obtained result to the original half space problem (1.1)–(1.3) in order to know the profiles of the solution $u(t,x)$ as $t \to +\infty$. The restriction to the one