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Coincidence of extendible vector-valued ideals with their minimal kernel

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ABSTRACT

We provide coincidence results for vector-valued ideals of multilinear operators. More precisely, if $\mathfrak{A}$ is an ideal of n-linear mappings we give conditions for which the equality $\mathfrak{A}(E_1, \dots, E_n; F) = \mathfrak{A}^{min}(E_1, \dots, E_n; F)$ holds isometrically. As an application, we obtain in many cases that the monomials form a Schauder basis of the space $\mathfrak{A}(E_1, \dots, E_n; F)$. Several structural and geometric properties are also derived using this equality. We apply our results to the particular case where $\mathfrak{A}$ is the classical ideal of extendible or Pietsch-integral multilinear operators. Similar statements are given for ideals of vector-valued homogeneous polynomials.

0. Introduction

A natural question in the theory of multilinear operators, and also of homogeneous polynomials, is to find conditions under which an ideal $\mathfrak{A}$ coincides isometrically with its minimal kernel $\mathfrak{A}^{min}$ (see [2,3,9,14,17,36] and [22, Section 33], which deal with problems of this nature). The reason for this is that, in many cases, this allows a tensorial representation of the ideal. Frequently, the tensor product inherits many structural characteristics from those properties of the spaces involved. For example, a known result due to Gelbaum and Gil de Lamadrid [31,32] states that the tensor $E_1 \tilde{\otimes}_\alpha E_2$ has a Schauder basis if both spaces E_1 and E_2 have a basis. This can be extended recursively to tensor products of any number of spaces, since the tensor product is associative (see the comments before Theorem 2.10 below).

Other properties (such as separability, Asplund or the Radon–Nikodým properties), in many cases are also preserved by the tensor product (see for example [10–12,17,40] and also the references therein). A tensorial representation of the ideal and these kinds of transference results, permit to deduce many attributes of the

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space $\mathfrak{A}(E_1, \ldots, E_n; F)$. Moreover, as the elements of $\mathfrak{A}^{min}$ may be usually approximated by finite type operators, we obtain the same property for $\mathfrak{A}$.

Therefore, we are interested in knowing when the canonical mapping

$$\varrho : \left(E_1' \,\tilde{\otimes} \ldots \tilde{\otimes}\, E_n' \,\tilde{\otimes}\, F; \alpha\right) \xrightarrow{1} \mathfrak{A}^{min}(E_1, \ldots, E_n; F) \hookrightarrow \mathfrak{A}(E_1, \ldots, E_n; F)$$

is a quotient mapping or an isometric isomorphism (here α stands for the tensor norm associated to $\mathfrak{A}$). Obviously, in both cases, we get $\mathfrak{A}^{min}$ equal to $\mathfrak{A}$.

Lewis ([36] and [22, 33.3]) obtained many results of the form $\mathcal{A}^{min}(E; F') = \mathcal{A}(E; F')$ if $\mathcal{A}$ is a maximal operator ideal or, in other words, coincidences of the form $E' \,\tilde{\otimes}_\alpha\, F' = (E \,\tilde{\otimes}_{\alpha'}\, F)'$. Based on Lewis' work, the first author and Carando tackled in [17] an analogous problem for scalar ideals of multilinear operators and polynomials (i.e., where the target space is the scalar field). As in Lewis' theorem, the Radon–Nikodým property becomes a key ingredient. In this article we follow the lines of [17] to address the vector-valued case. We stress that the vector-valued problem adds some technical difficulties.

For a given ideal of multilinear operators, we introduce in Definition 2.1 a vector-valued Radon–Nikodým property in the sense of [36,17] (where a similar property is given for tensor norms). This definition is related to a coincidence result in c_0-spaces. For extendible ideals (ideals in which every multilinear operator can be extended to any superspace of the domain, see Section 1) which enjoy the latter property we prove, in the main theorem, Theorem 2.4, that $\mathfrak{A}^{min}(E_1, \ldots, E_n; F)$ coincides isometrically with $\mathfrak{A}(E_1, \ldots, E_n; F)$ for Asplund spaces $E_1, \ldots, E_n$. It is noteworthy that the main theorem is not only based on what was done by Lewis and Carando–Galicer, but also generalizes both results. Finally we relate in Theorem 2.10, Proposition 2.11 and Theorem 2.12, intrinsic attributes of $\mathfrak{A}(E_1, \ldots, E_n; F)$ with properties of $E_1, \ldots, E_n, F$ and their tensor product. Namely, the existence of Schauder bases, separability, the Radon–Nikodým and Asplund properties.

We give some applications of these results for the ideals of extendible multilinear operators $\mathcal{E}$ and Pietsch-integral multilinear operators $\mathcal{PI}$. In Corollary 3.2, we obtain that if $E_1, \ldots, E_n$ are Asplund spaces and F' contains no copy of c_0 then the canonical mapping between $\mathcal{E}^{min}(E_1, \ldots, E_n; F')$ and $\mathcal{E}(E_1, \ldots, E_n; F')$ is an isometric isomorphism. Moreover, if $E_1', \ldots, E_n', F'$ also have a basis, then the monomials with the square ordering form a Schauder basis of $\mathcal{E}(E_1, \ldots, E_n; F')$. With additional hypothesis we obtain, in Corollary 3.3, similar statements in the case where the target space is not necessarily a dual space. In Corollary 3.5 we prove that $\mathcal{E}(E_1, \ldots, E_n; F')$ has the Radon–Nikodým property if and only if $E_1, \ldots, E_n, F$ are Asplund spaces.

A classical result due to Alencar [2] shows that the space of Pietsch-integral multilinear operators $\mathcal{PI}$ coincides isometrically, on Asplund spaces, with its minimal kernel (the space of nuclear operators, $\mathcal{N}$). We deduce this statement as a particular case of our main results. It is worth mentioning that we do not use the vector measure theory machinery [23] as Alencar did. Our perspective is completely different, it strongly relies on tensor techniques and the fact that $\mathcal{PI}$ enjoys the vector-valued Radon–Nikodým property. We obtain in Corollary 3.8 a coincidence result for the ideal $\mathcal{GI}$ of Grothendieck-integral multilinear operators as well.

We also state coincidence results for vector-valued ideals of homogeneous polynomials. In particular, we obtain in Section 4 similar theorems for $\mathcal{P}_e$ and $\mathcal{P}_{\mathcal{PI}}$ (i.e., the ideals of extendible and Pietsch-integral homogeneous polynomials, respectively). For the latter ideal, we recover some known results given in [9,14].

The article is organized as follows. In Section 1 we state all the necessary background on ideals of multilinear operators and their associated tensor norms. We also fix some standard notation and recall basic definitions of the theory of Banach spaces. In Section 2 we prove our vector-valued Lewis type theorems (coincidence results) and some of their consequences. In Section 3 we give the mentioned applications for the ideals $\mathcal{E}$ and $\mathcal{PI}$. Finally, in the last section we extend our results to the polynomial context.