J. Math. Anal. Appl. 418 (2014) 638–675

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Well-posedness and time-decay for compressible viscoelastic fluids in critical Besov space

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A R T I C L E I N F O

Article history:
Received 19 March 2014
Available online 12 April 2014
Submitted by H.-M. Yin

Keywords:
Compressible viscoelastic fluids
L^p based Besov space
Global well-posedness
Time-decay

A B S T R A C T

In this paper, we construct local solution with highly oscillating initial velocity and then get the global strong solution in the L^p based Besov space which improves the result of J. Qian, Z. Zhang (2010) [25] and X. Hu, D. Wang (2011) [14]. The local existence and uniqueness lies on the Lagrange coordinate transform and the contraction mapping theorem. The global result lies on a decomposition of the system and some commutator estimates. In the last part, we prove a time-decay in the critical Besov space framework which seems to have little investigation. The proof is based on the properties of the Green's matrix and various interpolations between Besov type spaces.

1. Introduction

Many fluids do not satisfy the Newtonian law. A viscoelastic fluid of the Oldroyd type is one of the classical non-Newtonian fluids which exhibits elastic behavior, such as memory effects. The elastic properties of the fluid are described by associating the fluid motions with an energy functional of deformation tensor U. Let us assume the elastic energy is $W(U)$, then the compressible viscoelastic system can be written as

$$\begin{cases} \partial_t\rho + \operatorname{div}(\rho u) = 0, \\ \partial_t(\rho v) + \operatorname{div}(\rho v \otimes v) + \nabla P(\rho) = \operatorname{div}\big(2\mu\mathcal{D}(v) + \nabla\big(\lambda \operatorname{div}(v)\big)\big) + \operatorname{div}\left(\dfrac{W_U(U)U^T}{\det(U)}\right), \\ \partial_t U + u\cdot\nabla U = \nabla u U. \end{cases} \tag{1.1}$$

Here ρ is the density and $v(x,t)$ is the velocity of the fluid. The pressure $P(\rho)$ is a given state equation with $P'(\rho) > 0$ for any ρ and $\mathcal{D}(v) = \frac{1}{2}(\nabla v + \nabla v^T)$ is the strain tensor. The Lamé coefficients μ and λ are assumed to satisfy

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http://dx.doi.org/10.1016/j.jmaa.2014.04.008
0022-247X/

$$\mu > 0 \quad \text{and} \quad \lambda + 2\mu > 0. \tag{1.2}$$

Such a condition ensures ellipticity for the operator $-\nabla(2\mu\mathcal{D}\cdot) - \nabla(\lambda\nabla\cdot)$ and is satisfied in the physical case, where $\lambda + 2\mu/N \approx 0$. Moreover, $W_U(U)$ is the Piola–Kirchhoff tensor and $\frac{W_U(U)U^T}{\det(U)}$ the Cauchy–Green tensor, respectively. For a special case of the Hookean linear elasticity, $W(U) = |U|^2$.

For the incompressible viscoelastic fluids, there are many important works recently. In [16], the authors proved the well posedness problem and found the relation

$$\nabla_k F^{ij} - \nabla_j F^{ik} = F^{lj}\nabla_l F^{ik} - F^{lk}\nabla_l F^{ij},$$

with $F = U - I$. This relation indicates that the linear term $\nabla \times F$ is actually a higher order term. F. Lin, C. Liu and P. Zhang [19,20] proved the local well posedness in Hilbert space H^s, and global well posedness with small initial data. In the proof of the global part, they capture the damping mechanism on F through very subtle energy estimates. At last, in [26], the authors proved the global well posedness of the incompressible version of system (1.1) in the critical L^p framework which allows us to construct the unique global solution for highly oscillating initial velocity.

For compressible viscoelastic fluids, in [14,25] the authors proved the local and global well-posedness in the L^2 based critical Besov type space. Their work used the properties of the viscoelastic fluids deeply and their results indicated that the deformation tensor U plays a similar role as the density ρ. It should be mentioned that the global existence of a smooth solutions is still an open problem, even in for incompressible viscoelastic fluids. P. Lions and N. Masmoudi [21] proved the global existence of a weak solution with general initial data in the case that the contribution of the strain rate in the constitutive equation is neglected. So it is important to study the global existence of system (1.1) near equilibrium in a functional space that is as large as possible.

There are two main goals: firstly, we want to establish the local and global well posedness in the L^p based critical Besov space. We use Lagrange coordinate method to obtain local existence and uniqueness with highly oscillating initial velocity. For global results, the main difficulty is that the stress tensor U only has weak dissipation which is revealed in [25] and [14]. In [25], the authors transformed the system into a more complex hyperbolic parabolic system and the Green's matrix is too complex to be studied. There are nineteen equations for the transformed system, so it is hard to generalize this method to the L^p based Besov space. In [14], X. Hu and D. Wang used a different idea. The authors decomposed the whole system into three similar subsystems. Since each subsystem can be handled easily, the authors can get the global existence. So it seems this method can be generalized to the L^p based Besov space framework. However, if we want to ensure the equivalence of transformed system and original system, we need to impose a condition on p which prevents us from getting the global existence with highly oscillating initial velocity. If we have the condition $\operatorname{div}(U^T) = 0$ as in the incompressible case, we may obtain the global existence with highly oscillating initial velocity. The more explicit results can be found in Theorem 2.4.

Secondly, we would like to give a time decay rate in the Besov space framework. Time decay is studied by many authors for the compressible Navier–Stokes equations [10,12,13,17,18,22,24]. For the compressible viscoelastic fluid, X. Hu and G. Wu in [15] give a detailed analysis about time-decay rate in the Sobolev space framework. However, in the Besov space framework, there is little work about time-decay rate for compressible fluid system even for the compressible Navier–Stokes equations. Our second aim is to get a time-decay rate as in [1] which gives a time decay of incompressible Navier–Stokes equations in the critical Besov space framework. Under low regularity assumptions (compared with H^3 used in compressible Navier–Stokes equations), we get that the solution will decay in the critical framework. The explicit results can be found in Theorem 2.4.

This paper will be organized as follows: In Section 2, we give some basic properties of the viscoelastic fluids and the main results. In Section 3, we will give a short introduction to the Littlewood–Paley decomposition and Besov space. In Section 4, we give the proof of local existence and a continuation criterion which is