



On a piecewise-smooth map arising in ecology



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ABSTRACT

In this paper, we study a two-dimensional piecewise smooth map arising in ecology. Such map, containing two parameters d and β , is derived from a model describing how masting of a mature forest happens and synchronizes. Here d is the energy depletion quantity and β is the coupling strength. Our main results are the following. First, we obtain a “weak” Sharkovskii ordering for the map on its nondiagonal invariant region for a certain set of parameters. In particular, we show that its Sharkovskii ordering is the natural number (resp., the positive even number) for $\beta > 1$ (resp., $0 < \beta < 1$). Second, we obtain a region of parameter space for which its corresponding global dynamics can be completely characterized.

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1. Introduction

Many dynamical systems contain terms that are non-smooth functions of their arguments. Important examples are electrical circuits having switches, mechanical devices in which components make impact on each other, problems with friction, sliding or squealing [4]. Even one-dimensional piecewise-smooth maps are known to have surprisingly rich dynamics, including periodic orbits with high period and the period-adding bifurcations [9,10]. In this paper, we are to investigate a certain properties and the global dynamics of a piecewise-smooth map arising in ecology. Specifically, we are concerned with a two dimensional piecewise-smooth map $F_{d,\beta}$ with two positive parameters d and β of the form:

$$\begin{aligned} F_{d,\beta}(x, y) &= (x + 1 - (d[y]_+^\beta + 1)[x]_+, y + 1 - (d[x]_+^\beta + 1)[y]_+) \\ &=: (F_1(x, y), F_2(x, y)), \end{aligned}$$

where $1 - d \leq x, y \leq 1$, $d, \beta > 0$, and $[x]_+ = x$ if $x \geq 0$; $[x]_+ = 0$ if $x < 0$. (1.1)

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We then define its associated lower dimensional map $f_{d,\beta}$ to be of the form:

$$f_{d,\beta}(x) = \begin{cases} x + 1 =: f_1(x), & \text{if } x \leq 0, \\ -dx^{\beta+1} + 1 =: g_{d,\beta}(x), & \text{if } x > 0. \end{cases} \tag{1.2}$$

It should be remarked that the diagonal set S , $S = \{(x, y): x = y, 1 - d \leq x, y \leq 1\}$, is invariant under the map $F_{d,\beta}$. In fact, $f_{d,\beta}(x)$ is obtained from $F_{d,\beta}$ by restricting its dynamics on S . Hence, we shall call $f_{d,\beta}$ the synchronous map of the system. Let $I_d = [1 - d, 1]$ and $D_d = I_d \times I_d$. Then all initial iterates of $f_{d,\beta}$ and $F_{d,\beta}$ enter the invariant region I_d and D_d , respectively, at finite time. We shall, henceforth, treat the domains of $f_{d,\beta}$ and $F_{d,\beta}$ to be I_d and D_d , respectively. The ecological meaning of the parameters d and β are to be explained in Section 2.

2. Forest model

To give some ideas as what model gives rise to (1.1) and (1.2), we shall begin with a brief introduction of the coupled trees model considered by Isagi et al. [5], and Satake and Iwasa [11]. Let $Y(k)$ be the amount of normalized energy reserve at the beginning of year k . Here $Y(k)$ is normalized in a way that 0 is the critical energy level for a tree to reproduce and that its energy level lies in between 1 and $-d + 1$, where d is the depletion coefficient. Due to the photosynthesis, each tree will gain a normalized amount of energy each year. Moreover, if the energy level of the tree is below critical at the year k , then all its energy is preserved to the following year. On the other hand, if its energy exceeds the critical level, then it will set flower and grow seeds. As a result, its energy is decreased after a reproductive year. The quantity d is a measurement to such energy depletion. Consequently, the motion of its energy reserve yearly for an individual tree reads as follows.

$$Y(k + 1) = \begin{cases} Y(k) + 1 & \text{if } Y(k) \leq 0, \\ -dY(k) + 1 & \text{if } Y(k) > 0. \end{cases} \tag{2.1}$$

If the resource depletion by fruit production is heavy, meaning a larger d , the reproductive activities tend to fluctuate between years with a large variance. In a mature forest, fruiting efficiency may depend on the flowering activity of the other trees in a forest. This is because the pollination efficiency changes with the number of plants flowering in a population. To model the pollen limitation of the reproduction, d in (2.1) is replaced by $dP_i(k)$, where $P_i(k)$ is a factor smaller than or equal to 1, and indicates outcross pollen availability for the i -th tree. Then the normalized energy reserve of the i -th tree at year $k + 1$ is

$$Y_i(k + 1) = \begin{cases} Y_i(k) + 1 & \text{if } Y_i(k) \leq 0, \\ -dP_i(k)Y_i(k) + 1 & \text{if } Y_i(k) > 0, \end{cases} \tag{2.2a}$$

where

$$P_i(k) = \left(\frac{1}{n - 1} \sum_{\substack{j=1 \\ j \neq i}}^n [Y_j(k)]_+ \right)^\beta. \tag{2.2b}$$

Here n is the total number of trees in the forest and β is the coupling strength, which measures the efficiency of the spread of outcross pollen produced by other flowering activities. In fact, the rate of setting seeds and fruits is limited by its pollen availability, which depends on the coupling strength β . The coupling strength β determines the shape of the outcross pollen availability function $P_i(k)$ and controls the degree of dependence of fruit production on $P_i(k)$. If β is chosen to be closed to zero, then the fruit production is almost independent of the reproductive activities of the other trees in a forest. Small β means that a small

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