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Exact boundary controllability of vibrating non-classical Euler–Bernoulli micro-scale beams

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ABSTRACT

This study investigates the exact controllability problem for a vibrating non-classical Euler–Bernoulli micro-beam whose governing partial differential equation (PDE) of motion is derived based on the non-classical continuum mechanics. In this paper, it is proved that via boundary controls, it is possible to obtain exact controllability which consists of driving the vibrating system to rest in finite time. This control objective is achieved based on the PDE model of the system which causes that spillover instabilities do not occur.

1. Introduction

These days, micro-scale beams represent a major structural component and play a significant role in many micro- and nano-electromechanical (MEMS and NEMS) devices such as in mechanical resonators [21], micro-switches [8], vibration sensors [17], atomic force microscopes (AFM) [7] and various actuators [5].

The beam thickness is typically on the order of microns and sub-microns in various applications of MEMS based micro-beams. To capture the size dependent elastic behavior of materials at small scales, conventional continuum mechanics requires to be extended by using higher order continuum theories. This fact has been proven by a number of experimental observations such as those in references [6,18,29] by investigating the size effect in micro-scale beams.

One of the most successful and inclusive higher-order non-classical continuum theories is modified strain gradient theory elaborated by Lam et al. in 2003 [13]. Recently, using this theory, many research publications have been devoted to study the static analysis, dynamic modeling and vibration analysis of non-classical micro-beams. For example, in 2009, the new governing equations of equilibrium and corresponding boundary conditions were derived for non-classical Euler–Bernoulli micro-beams by using a combination

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of the modified strain gradient theory and the Hamilton principle, and static and dynamic problems of them are investigated [11]. In 2012, Zhao et al. [35] established nonlinear governing equation of strain gradient Euler–Bernoulli micro-scale beams and investigated nonlinear free vibration of them. Vatankhah et al. [30] studied nonlinear forced-vibration of non-classical strain gradient Euler–Bernoulli micro-beams in 2013.

The necessity of existing high performance control systems for improving the performance of micro-scale instruments based on micro-beam structures caused that many investigators have been conducted in vibration control of classical micro-beams. Although vibration control of non-classical micro-beams is a significant subject in engineering applications, but almost all of the publications in the field of non-classical micro-beams have been devoted to the static and dynamic properties and vibration analysis in recent years. In this regard, authors studied asymptotic stabilization of non-classical strain gradient micro-beams using boundary controllers in 2013 [31].

The present investigation considers the exact controllability of the governing partial differential equation (PDE) of non-classical strain gradient free–free micro-beams using boundary actions. Using boundary actuators is a superior approach especially from applied and engineering points of view; because we deal only with the actuators along the boundaries and using in-domain ones is not necessary. Also, control design based on the PDE model causes that spillover instabilities, which appear when the control design is accomplished for the finite dimensional model of the system, do not occur. In this study, the main analytical tools for contemplating the exact controllability problem are semigroup techniques and Hilbert Uniqueness Method (HUM).

2. Dynamic model of non-classical strain gradient micro-beams

To arrive at the partial differential equation (PDE) of motion and corresponding boundary conditions (BCs) of the linear strain gradient Euler–Bernoulli micro-scale beam, one can apply a combination of the modified strain gradient theory and the Hamilton principle, and obtain the following results [11]:

$$\rho A\frac{\partial^2 y}{\partial t^2}+K_1\frac{\partial^4 y}{\partial x^4}-K_2\frac{\partial^6 y}{\partial x^6}=0 \tag{1}$$

$$\begin{cases} \left(K_1\dfrac{\partial^3 y}{\partial x^3}-K_2\dfrac{\partial^5 y}{\partial x^5}+F\right)\Big|_{x=0,L}=0 \text{ or } \delta y|_{x=0,L}=0, \\ \left(K_2\dfrac{\partial^4 y}{\partial x^4}-K_1\dfrac{\partial^2 y}{\partial x^2}+M^c\right)\Big|_{x=0,L}=0 \text{ or } \delta\dfrac{\partial y}{\partial x}\Big|_{x=0,L}=0, \\ \left(K_2\dfrac{\partial^3 y}{\partial x^3}-M^{n.c}\right)\Big|_{x=0,L}=0 \text{ or } \delta\dfrac{\partial^2 y}{\partial x^2}\Big|_{x=0,L}=0, \end{cases} \tag{2}$$

where L, A and ρ illustrate the length, uniform cross-section area and density of the beam, respectively, x and t indicate the independent spatial and time variables, respectively, $y(x,t)$ represents the lateral deflection, F demonstrates the resultant transverse forces in a section caused by the classical stress components acting on the section, M^c is the resultant moment in a section caused by the classical and higher-order stress components and $M^{n.c}$ refers to the higher-order non-classical resultants in a section caused by higher-order stresses acting on the section. In addition,

$$\begin{aligned} K_1&=EI+\mu A\left(2l_0^2+\frac{43}{225}l_1^2+l_2^2\right) \\ K_2&=\mu I\left(2l_0^2+\frac{4}{5}l_1^2\right) \end{aligned} \tag{3}$$

in which I is the area moment of inertia of the beam cross-section and E is the Young modulus. Also, l_0, l_1 and l_2 appeared in coefficients K_1 and K_2, demonstrate the additional independent material parameters.