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Null controllability of a pseudo-parabolic equation with moving control

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ABSTRACT

In this paper, we study the internal controllability of the pseudo-parabolic equation on the one-dimensional torus. Our control function is acting on a moving small interval with a constant velocity. With this moving distributed control, we obtain the system is null controllable for given data with certain regularity.

1. Introduction

This paper is devoted to studying controllability properties of the pseudo-parabolic equation:

$$\begin{cases} y_t - y_{xx} + y - \kappa y_{xxt} = \mathbf{1}_{\omega(t)} u(x,t), \\ y(x,0) = y_0(x), \end{cases} \tag{1.1}$$

where t is time, $x \in \mathbb{T} = \mathbb{R}/(2\pi\mathbb{Z})$ is the space variable, $\kappa > 0$ is a constant, u is a control function and $\mathbf{1}_{\omega(t)}$ stands for the characteristic function of the set $\omega(t)$ that constitutes the support of the control, localized in a moving subset $\omega(t)$ of $\mathbb{T}$.

The pseudo-parabolic equations are a kind of Sobolev–Galpern type equations. They have occurred in numerous physical applications among which include problems involving seepage of fluids through fissured rocks [1] (where κ is a characteristic of the fissured rock), unsteady flows of second-order fluids [6,18], the theory of thermodynamics involving two temperatures [5]. They can also be used as a regularization of ill-posed transport problems, especially as a quasi-continuous approximation to discrete models for population dynamics [13].

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Our goal in this paper is to prove the null controllability of (1.1) with moving control. We say that the system is null controllable at time T if for any initial data y_0, there exists a corresponding control function u such that $y(\cdot, T) = 0$.

It is known that pseudo-parabolic equations are closely related to the well-known Benjamin–Bona–Mahony (BBM) equations

$$y_t + y_x - y_{xxt} + yy_x = 0. \tag{1.2}$$

In [11], Micu showed the linearized BBM equation fails to be spectrally controllable (hence not null controllable) with a control supported in a fixed domain. Such a phenomenon was also proved for wave equation with structural damping [15]. Similarly, in [17], we found that as the third order term arises, with a boundary control, the pseudo-parabolic equation is not null controllable, but the approximate controllability result can be obtained. In fact, the bad control properties of these equations come from the BBM term y_{txx} which causes the eigenvalues of the equations to have at least one finite accumulation point. It is worth mentioning that if $\omega(t) \equiv \omega$, i.e. the support of the control does not move in time as it is often considered, the pseudo-parabolic equation is not null controllable. This property can be obtained by the standard smooth extension method since the pseudo-parabolic equation is not null controllable with a boundary control. A similar result was pointed in [12].

In order to obtain a better control property for the equations with the BBM term y_{txx}, the moving control could be an available option. The concept of moving point control was introduced by J.L. Lions in [9]. And the controllability of equations with a moving point control was investigated in [2,3,7–9]. Recently, Rosier and Zhang [16] introduced the idea of moving control into the BBM equation. They set $z(x,t) = y(x - ct, t)$ with $c \in \mathbb{R}$ and then transformed (1.2) into a KdV-BBM equation:

$$z_t + (c+1)z_x - cz_{xxx} - z_{txx} + zz_x = 0. \tag{1.3}$$

Thanks to the KdV term $-cz_{xxx}$, they got that (1.3) with a moving distributed control is exactly controllable in large time. A similar conclusion was proved in [10] for the structurally damped wave equation. Motivated by the above works, as the bad control properties of (1.1) come from the third order BBM term $-\kappa y_{txx}$, it is natural to ask whether better control properties for (1.1) could be obtained by using a moving control. The aim of this paper is to investigate that issue.

For the sake of simplicity, we will take $\kappa = 1$ throughout this paper. All the results can be extended without difficulty to $\kappa > 0$ arbitrary. Suppose y solve

$$y_t - y_{xx} + y - y_{xxt} = b(x + ct)u(x,t). \tag{1.4}$$

Let $z(x,t) = y(x - ct, t)$, then z satisfies

$$z_t - z_{xx} + z + cz_x - z_{xxt} - cz_{xxx} = b(x)u(x - ct, t), \tag{1.5}$$

with $z(x, 0) = y_0(x)$. This transformation brings us the KdV term $-cz_{xxx}$, which causes that there is no accumulation point in the spectrum with $u = 0$ in (1.5) and this property makes the controllability could be hold probably. Therefore, we will use this good property to achieve the desired null controllability with moving control. The key observation of our approach is to construct a suitable family $\{\psi_k\}_{k\in\mathbb{Z}}$ of entire functions of exponential type, by which we can solve the null controllability problem after turning it into a moment problem.

Throughout this paper, we assume that $c = -1$, while for any $c \neq 0$ can be derived in the same way and we denote H^s the classical Fourier definition of Sobolev spaces. Our main results are stated as follows. Consider the control problem