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Existence of traveling wave solutions for influenza model with treatment

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ABSTRACT

To investigate the spreading speed of influenza and the influence of treatment on the spreading speed, a reaction–diffusion influenza model with treatment is established. The existence of traveling wave solutions is shown by introducing an auxiliary system and applying the Schauder fixed point theorem. The non-existence of traveling wave solutions is proved by a two-sided Laplace transform, which needs a new approach for the prior estimate of exponential decay of traveling wave solutions.

1. Introduction

Influenza is a contagious respiratory illness caused by influenza viruses. It can cause mild to severe illness, and at times leads to death [5,7]. Influenza spreads around the world in seasonal epidemics, resulting in about three to five million yearly cases of severe illness and about 250,000 to 500,000 yearly deaths [30].

Several mathematical models have been proposed to understand influenza dynamics and the influences of prevention and control measures, which include vaccination, antiviral use, isolation and prophylaxis [1–3,7,8,10,15,16,18,20,21]. Usually, an infectious case is first found at one location and then the disease spreads to other areas [22]. Consequently, an important question for influenza is: what is the spreading speed? Traveling wave solution is an important tool used to study the spreading speed of influenza [14,23,17,24,25,31]. However, traveling waves in these models are found by numerical simulations and the relations between the minimum wave speed and the parameters are not apparent. In this paper, we incorporate the population diffusions into the epidemic influenza model with the treatment in [15] and give conditions for the existence and non-existence of traveling wave solutions.

We divide the total density of humans at location x and time t into four subclasses: susceptible $S(t,x)$, infected and untreated $I_u(t,x)$, treated $I_h(t,x)$, and recovered $R(t,x)$. Neglecting prophylaxis and antiviral

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resistance and incorporating diffusions of human individuals into the model of [15], we obtain the following mathematical model:

$$\begin{cases} \dfrac{\partial S}{\partial t} = d_s \dfrac{\partial^2 S}{\partial x^2} - \beta(I_u + \delta I_h)S, \\ \dfrac{\partial I_u}{\partial t} = d_u \dfrac{\partial^2 I_u}{\partial x^2} + (1-\mu)\beta(I_u + \delta I_h)S - k_u I_u, \\ \dfrac{\partial I_h}{\partial t} = d_h \dfrac{\partial^2 I_h}{\partial x^2} + \mu\beta(I_u + \delta I_h)S - k_h I_h, \\ \dfrac{\partial R}{\partial t} = d_r \dfrac{\partial^2 R}{\partial x^2} + k_u I_u + k_h I_h, \end{cases} \tag{1}$$

where δ is the reduction factor in infectiousness due to the antiviral treatment, β stands for the transmission coefficient of untreated infected individuals, μ denotes the fraction of new infected cases who are treated, k_u and k_h are the recovery rates for untreated cases and treated cases, respectively, and d_s, d_u, d_h and d_r are the corresponding diffusion coefficients for the four subclasses. By biological meaning we have $0 < \mu < 1$, $0 < \delta < 1$ and $k_h > k_u$.

Asymptotic spreading speed is an important concept which describes the diffusive speed of populations. For a cooperative system, the linear determinacy may be available to calculate the asymptotic spreading speed [29]. However, it is generally difficult to obtain its existence for a non-cooperative system. Li, Weinberger and Lewis [12] believe that the asymptotic spreading speed can be characterized as the lowest speed of traveling wave solutions (minimal wave speed). Thus, one solution for finding asymptotic spreading speed is to examine the existence of minimal wave speed and to show that minimal wave speed is equal to asymptotic spreading speed by numerical simulations. In this paper, we will figure out the existence of minimal wave speed for model (1), which is an important step towards the asymptotic spreading speed.

The Schauder fixed point theorem is an important method for the existence of traveling wave solutions, which is applied widely (for example, [6,11,13,26–28]). Since system (1) consists of more than two equations, we introduce an auxiliary system by which a bounded cone is constructed so that the Schauder fixed point theorem can be applied. Motivated by [4,26–28], we prove the non-existence of traveling wave solutions by a two-sided Laplace transform, which was firstly introduced by Carr [4] and then was applied by Wang et al. [26–28]. The application of the two-sided Laplace transform needs the prior estimate of exponential decay of traveling wave solutions (see, for example, [28]). We point out that the analytical method in [28] cannot be applied to our model since model (1) consists of more than two equations. Because of this reason, we propose a new method in this paper to get the prior estimate of exponential decay of traveling wave solutions of (1), which is inspired by the proof of Stable Manifold Theorem in [19] and the invariance of traveling wave solutions under translation. It seems that this method can be applied to more general reaction–diffusion systems consisting of more than two equations.

This paper is organized as follows. Next section is devoted to the existence of traveling wave solutions. Firstly, an auxiliary system is introduced to prove the existence of traveling wave solutions and the minimal wave speed is studied by the linearization. Then a positively invariant cone is constructed by a pair of upper and lower solutions so that the Schauder fixed point theorem can be applied. Finally, the existence of traveling wave solutions of (1) is obtained by limiting arguments. Section 3 focuses on the non-existence of traveling wave solutions. A new method is proposed to get the prior estimate of exponential decay of traveling wave solutions and two-sided Laplace transform is applied to prove the non-existence of traveling wave solutions. Section 4 presents brief simulations and discussions.