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Existence of homoclinic solutions for higher-order periodic difference equations with p-Laplacian ☆

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ABSTRACT

Using the critical point theory in combination with periodic approximations, we establish sufficient conditions on the existence of homoclinic solutions for higher-order periodic difference equations with p-Laplacian. Our results provide rather weaker conditions to guarantee the existence of homoclinic solutions and considerably improve some existing ones even for some special cases.

1. Introduction and main results

The aim of this paper is to study the existence of homoclinic solutions for a class of periodic difference equations

$$(-1)^n\Delta^n\big[r(k)\phi_p\big(\Delta^n u(k-n)\big)\big]+q(k)\phi_p\big(u(k)\big)=f\big(k,u(k)\big)\quad \text{for all } k\in Z, \tag{1.1}$$

where n is a fixed positive integer, $p>1$ is a fixed real number and $\phi_p(t)=|t|^{p-2}t$ for all $t\in R$. $q(k)$ and $r(k)$ are positive and T-periodic sequences, T is a fixed positive integer. $f: Z\times R\to R$ is continuous in the second variable and T-periodic in the first variable. Moreover, Δ is the forward difference operator defined by $\Delta u(k)=u(k+1)-u(k)$, $\Delta^2 u(k)=\Delta(\Delta u(k))$.

Difference equations, the discrete analog of differential equations, have been widely used in many fields such as computer science, economics, neural network, ecology, cybernetics, etc. [1]. In the past decade,

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the existence of periodic, subharmonic and homoclinic solutions for difference equations has been extensively studied, to mention a few, see [2–14,16–20,22–28]. Among the methods used are critical point theory, fixed point theory and Nehari manifold approach and so on.

In the aforementioned references, most of the difference equations involved are lower order. In particular, Ma and Guo [13,14] and Iannizzotto and Tersian [9] provided some sufficient conditions on the existence of homoclinic solutions for second-order difference equations. Recently, there has been an increasing interest in higher-order difference equations [4–6,21,25,28]. In Deng et al. [6], the periodic and subharmonic solutions for a higher-order periodic difference equation with p-Laplacian have been studied. However, to our best knowledge, it seems that no similar results about the homoclinic solutions of higher-order periodic difference equations with p-Laplacian are obtained in the literature. Many papers (see e.g. [13,14]) considered the nontrivial homoclinic solutions of second-order difference equations, they usually assumed that the Ambrosetti–Rabinowitz condition was satisfied:

(**AR**) there exists a constant $\beta > 2$ such that

$$uf(k,u) \geqslant \beta F(k,u) > 0 \quad \text{for all } k \in Z \text{ and } u \in R\backslash\{0\}.$$

It is well known that many nonlinearities such as

$$f(k,u) = u^3\left(1+u^2\right)^{-1}$$

don't satisfy (**AR**). Therefore, in this paper, we will consider the existence of homoclinic solutions for higher-order periodic difference equations involving p-Laplacian without the generalized Ambrosetti–Rabinowitz condition:

(**GAR**) there exists a constant $\beta > p$ such that

$$uf(k,u) \geqslant \beta F(k,u) > 0 \quad \text{for all } k \in Z \text{ and } u \in R\backslash\{0\}.$$

As usual, a solution $u = \{u(k)\}$ of (1.1) is said to be homoclinic (to 0) if

$$\lim_{|k|\to+\infty} u(k) = 0.$$

In addition, if $u(k) \equiv 0$ for $k \in Z$, then u is called the trivial solution, otherwise u is called a nontrivial homoclinic solution. Obviously, the nontrivial solution in l^p of (1.1) is the nontrivial homoclinic solution of (1.1).

Throughout this paper, we always assume that the following conditions are satisfied:

(**r**) $r(k) > 0$ and $r(k+T) = r(k)$ for all $k \in Z$.
(**q**) $q(k) > 0$ and $q(k+T) = q(k)$ for all $k \in Z$.
(**f**) $f(k,u)$ is continuous in u and T-periodic in k, and $F(k,u) = \int_0^u f(k,s)\,ds$ for $u \in R$.

By periodicity of $\{r(k)\}$ and $\{q(k)\}$ in k, their maximums and minimums can be achieved. Let r^* and r_* denote the maximum and minimum of $\{r(k)\}$ respectively. Similarly, let q^* and q_* denote the maximum and minimum of $\{q(k)\}$ respectively. By equivalence of $\|\cdot\|_p$ and $\|\cdot\|_2$ for finite dimensional space, there exists a constant $c > 0$ such that

$$\left(|u|^2+|v|^2\right)^{\frac{1}{2}} \leqslant c\left(|u|^p+|v|^p\right)^{\frac{1}{p}} \quad \text{for all } u,v \in R. \tag{1.2}$$

Now we are in a position to state our main results.