



# Global stability and wavefronts in a cooperation model with state-dependent time delay <sup>☆</sup>



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## ABSTRACT

This article deals with a diffusive cooperative model with state-dependent delay which is assumed to be an increasing function of the population density with lower and upper bounds. For the cooperative DDE system, the positivity and boundedness of solutions are firstly given. Using the comparison principle of the state-dependent delay equations obtained, the stability criterion of model is analyzed both from local and global points of view. When the diffusion is properly introduced, the existence of traveling waves is obtained by constructing a pair of upper–lower solutions and Schauder's fixed point theorem. Calculating the minimum wave speed shows that the wave is slowed down by the state-dependent delay. Finally, the traveling wavefront solutions for large wave speed are also discussed, and the fronts appear to be all monotone, regardless of the state dependent time delay. This is an interesting property, since many findings are frequently reported that delay causes a loss of monotonicity, with the front developing a prominent hump in some other delay models.

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## 1. Introduction

In a natural ecosystem, the maturity of a species individuals is not an instantaneous process but is mediated by some time lag which can be viewed as the time taken from birth to maturity. Systems with time lag (or time delay) lead to delay differential equations (DDE), which have been studied intensively and systematically [9,23,16,41]. The theory and applications of DDEs are emerging as an important area of investigation. Previously, some models of population growth with time delay (discrete and distributed time delays, stochastic, etc.) were discussed in literature [7,15,37,24,40].

But, in these above systems, only the constant time delay is considered. In 1992, Aiello et al. [2] have already considered a system with a state dependent delay, where the time delay is taken to be an increasing

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function of the total populations. This assumption is believed to be realistic in the example of Antarctic whale and seal populations [11]. It is observed that individual of a small seal species takes three to four years to mature and of large seals takes five years to mature, of small whales takes seven to ten years and of large whale species takes twelve to fifteen years to reach maturity. Besides, Andrewartha and Birch [6] considered how the duration of larval development of flies is viewed as a nonlinear increasing function of larval density. For the interesting phenomenon, many authors investigated state-dependent time delay population model in literature [43,45,1,21]. In fact, the state-dependent delay  $\tau(u_1)$  measures the intraspecific competition effects of a species  $u_1$ . Since the limited food resources made the species individuals devote more energy and time to finding food for their own survival and virtually none to reproduce, the time to maturity certainly becomes longer. That is, the period of maturity is longer if the number of species is larger, in return, it will lead to reduce the size of the population since the growth of the species is slowed down. Finally, the species will be equilibrium at some level  $u_1^*$ , and there is corresponding to an equilibrium delay  $\tau(u_1^*)$ . Our results imply that the stronger the intraspecific competition of the species, the smaller the equilibrium size  $u_1^*$  of the species and the lower the equilibrium delay  $\tau(u_1^*)$ . In this paper, we will deal with a diffusive cooperative model with state-dependent time delay. In biological terms, cooperation can be interpreted as that the presence of one species encourages the growth of the other species, which is one of the important interactions among species and is commonly seen in social animals and in human society (see, for example, [12,28,30]). Furthermore, we believe that such a diffusive model with state-dependent delay has not been discussed yet, and thus the work in this article is new.

As mentioned above, most species individuals have a life history that takes them through two stages: immature and mature, and species at two stages may have different behaviors. For example, for a number of mammals, the immature prey are concealed in the mountain cave and raised by their parents; they do not necessarily go out seeking food. When motion is allowed, then it is reasonable to suppose that the immature does not move (especially if the immature phase is a larval phase) and does not have a risk to contact with other species. Therefore, considering that stage structure in population is in accord with the natural phenomenon. Based on the fact that the amount of food available per biomass in a closed environment is a function of the consumer population, we propose the following diffusive cooperative model with a monotonically increasing, state-dependent delay

$$\begin{cases} \frac{dv_1}{dt} = \alpha_1 u_1 - \gamma_1 v_1 - \alpha_1 e^{-\gamma_1 \tau(u_1+v_1)} u_1 (t - \tau(u_1 + v_1)), \\ \frac{\partial u_1}{\partial t} = d_1 \frac{\partial^2 u_1}{\partial x^2} + \alpha_1 e^{-\gamma_1 \tau(u_1+v_1)} u_1 (t - \tau(u_1 + v_1)) - \beta_1 u_1^2 + \mu_1 u_1 u_2, \\ \frac{dv_2}{dt} = \alpha_2 u_2 - \gamma_2 v_2 - \alpha_2 e^{-\gamma_2 \tau(u_2+v_2)} u_2 (t - \tau(u_2 + v_2)), \\ \frac{\partial u_2}{\partial t} = d_2 \frac{\partial^2 u_2}{\partial x^2} + \alpha_2 e^{-\gamma_2 \tau(u_2+v_2)} u_2 (t - \tau(u_2 + v_2)) - \beta_2 u_2^2 + \mu_2 u_1 u_2, \end{cases} \quad t \in \mathbb{R}^+, \quad x \in \mathbb{R}. \quad (1.1)$$

The following assumptions for model (1.1) are made:

- The variables  $v_i(t, x)$  and  $u_i(t, x)$  ( $i = 1, 2$ ) represent the densities of the cooperative immature and mature species at time  $t$  and at position  $x$ , respectively.
- The parameter  $d_i (> 0)$ ,  $i = 1, 2$ , is diffusion coefficient of population  $u_i$ . The delay  $\tau$  is the time taken from birth to maturity. This paper considers the time delay to be state dependent, that is, the time delay is taken to be an increasing function of the total population  $u_i + v_i$ , so that  $\tau'(u_i + v_i) \geq 0$ , and we shall also assume that  $0 < \tau_m \leq \tau(u_i + v_i) \leq \tau_M$  with  $\tau_m = \tau(0)$  and  $\tau_M = \tau(\infty)$ .
- The rate at which individuals are born is taken to be proportional to the number of matures at that time; this is the  $\alpha_i u_i$  term. Death of immatures is modeled by the term  $-\gamma_i v_i$ . Death of matures is modeled by a quadratic term  $\beta_i u_i^2$ , as in the logistic equation. The term  $\alpha_i e^{-\gamma_i \tau(u_i+v_i)} u_i (t - \tau(u_i + v_i), x)$  appearing in both equations represents the rate at time  $t$  and position  $x$  at which individuals leave the immature

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