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Robust stability and stabilization of linear stochastic systems with Markovian switching and uncertain transition rates ☆

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ABSTRACT

This paper is concerned with the robust stabilization problem for a class of linear uncertain stochastic systems with Markovian switching. The uncertain stochastic system with Markovian switching under consideration involves parameter uncertainties both in the system matrices and in the mode transition rates matrix. New criteria for testing the robust stability of such systems are established in terms of bi-linear matrix inequalities (BLMIs), and sufficient conditions are proposed for the design of robust state-feedback controllers. A numerical example is given to illustrate the effectiveness of our results.

1. Introduction

Stochastic systems with Markovian switching have been used to model many practical systems where they may experience abrupt changes in their structure and parameters. Such systems have played a crucial role in many applications, such as hierarchical control of manufacturing systems [7,8,19], financial engineering [22] and wireless communications [10].

In engineering, stability is the most fundamental problem. Thus many authors have dealt with the problem. For example, [5] and [18] systematically studied stochastic stability properties of linear jump systems and the relationship among various moment and sample path stability properties. Refs. [1,2,17] discussed the stability in distribution of the random diffusion. Refs. [13] and [11] considered the exponential stability for general nonlinear stochastic differential equation with Markovian switching of the form

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$$dx(t) = f\big(x(t), t, r(t)\big)\,dt + g\big(x(t), t, r(t)\big)\,dB(t), \tag{1.1}$$

where $r(t)$ is a Markov chain taking values in $S = \{1, 2, \ldots, N\}$. Ref. [9] derived a sufficient condition for the p-stability for the regime-switching diffusion process. For some of the recent progress, we refer the reader to [14] and [23]. Over the last decade, stochastic control problems governed by stochastic differential equation with Markovian switching have attracted considerable research interest, and we here mention [4,12,24,27]. It is well known that uncertainty occurs in many dynamic systems and is frequently a cause of instability and performance degradation. Meanwhile, considerable attention has been given to the problem of designing robust controllers for linear systems with parameter uncertainty, such as [15,20,25,26].

It is obvious that the classical and powerful technique applied in the study of stability is the stochastic version of the Lyapunov direct method. Most of the existing conditions for stability implicitly require the assumption that the transition rates are completely known. In many situations, however, it is difficult to acquire the exact transition rates. Stabilization and stability of randomly switched systems without perfect knowledge of transition rates are nontrivial and important issue. Then, it leads to a study of the problem where the transition rates matrix $Q = (q_{ij})_{N\times N}$ is not precisely known.

This paper is concerned with the stochastic stabilization problem for a class of linear uncertain stochastic systems with Markovian switching. The uncertain system under consideration involves parameter uncertainties both in the system matrices and in the mode transition rates matrix. We aim at designing a robust state-feedback controller such that, for all admissible uncertainties, the closed-loop system is exponentially stable in mean square.

The structure of this paper is as follows. Section 2 describes some preliminaries. The main results are stated in Sections 3 and 4. A numerical example is given in Section 5 to illustrate the effectiveness of our results.

2. Preliminaries

In this paper, we will employ the following notation. Let $|.|$ be the Euclidean norm in $\mathbb{R}^n$. $\mathbb{R}_+$ is the interval $[0, \infty)$. If A is a vector or matrix, its transpose is denoted by A^T. I_n denotes the $n \times n$ identity matrix. If A is a symmetric matrix $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ mean the smallest and largest eigenvalues, respectively. If A and B are symmetric matrices, by $A > B$ or $A \geqslant B$ means that $A - B$ is positive definite or nonnegative definite. We write $diag(a_1, \ldots, a_n)$ for a diagonal matrix whose diagonal entries starting in the upper left corner are $a_1, \ldots, a_n$.

Let $(\Omega, \mathcal{F}, (\mathcal{F})_t, P)$ be a complete probability space with a filtration $(\mathcal{F})_t$ satisfying the usual conditions. Let $r(t), t \geqslant 0$, be a Markov chain on the probability space taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with generator $Q = (q_{ij})_{N\times N}$ given by

$$P\big(r(t+\Delta) = j \,\big|\, r(t) = i\big) = \begin{cases} q_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + q_{ii}\Delta + o(\Delta), & \text{if } i = j \end{cases}$$

where $\Delta > 0$, and $q_{ij} \geqslant 0$ denotes the transition rate from i to j if $i \neq j$ while $q_{ii} = -\sum_{i\neq j} q_{ij}$.

Consider the following stochastic differential equations with Markovian switching

$$\begin{aligned} &dx(t) = f\big(x(t), r(t)\big)\,dt + g\big(x(t), r(t)\big)\,dB(t), \\ &x(t_0) = x_0 \in R^n, \quad t \geqslant t_0, \end{aligned} \tag{2.2}$$

where $f : \mathbb{R}^n \times S \longmapsto \mathbb{R}^n$, $g : \mathbb{R}^n \times S \longmapsto \mathbb{R}^{n\times d}$ and $B(t)$ denotes a d-dimensional Brownian motion defined on the underlying probability space and independent of $r(t)$. Both f and g satisfy the local Lipschitz condition and grow at most linearly, and these conditions insure Eq. (2.2) has a unique solution (see [11,14]). We also assume that $f(0, i) = g(0, i) = 0$ for each $i \in S$. As a result, Eq. (2.2) admits a trivial solution $x(t, 0) = 0$.