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Asymptotic stability of a mathematical model of cell population

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ABSTRACT

We consider a simplified system of a growing colony of cells described as a free boundary problem. The system consists of two hyperbolic equations of first order coupled to an ODE to describe the behavior of the boundary. The system for cell populations includes non-local terms of integral type in the coefficients. By introducing a comparison with solutions of an ODE's system, we show that there exists a unique homogeneous steady state which is globally asymptotically stable for a range of parameters under the assumption of radially symmetric initial data.

1. Introduction

The growth of cells colonies has been a focus of research since the first studies of cancer. With the increasing number of experimental data have appeared a large hierarchy of mathematical models. In this paper, we consider a simplified mathematical model in a free boundary domain. The system simplifies the problem proposed in [9] based on a continuous distribution of cell population and modeled by hyperbolic equations.

Systems of hyperbolic equations in that context have been used before by several authors (see for instance Friedman [2,1], Friedman and Tao [3], Tao [8] and references therein).

We denote by "s" the density of a colony of cells in a bounded domain $\Omega(t)$ with moving boundary "$\partial\Omega(t)$". We assume that the cells die at constant death rate k_s. The dead cells are removed at constant rate k_d as they decompose at rate k_d' assumed constant. For technical reasons we take

$$k_d > \frac{4}{3}. \tag{1.1}$$

The volume is composed by the living cells and death cells and it is exemplified as a porous medium with a radial distribution in a spherical domain. The flow velocity in the interior of the colony is denoted by "v"

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and the density of death cells by d. After renormalization we may assume

$$k_d' = k_d$$

and the problem is described by the following system of equations

$$\begin{aligned} &\frac{\partial s}{\partial t} + \nabla \cdot (vs) = k_s s - k_d s, \quad 0 < t < T,\ x \in \Omega(t), \\ &\frac{\partial d}{\partial t} + \nabla \cdot (vd) = k_d s - k_d d, \quad 0 < t < T,\ x \in \Omega(t) \end{aligned} \tag{1.2}$$

with regular initial data.

The growth factor depends of the total mass of living cells and it is defined by

$$k_s = 1 - \frac{1}{|\Omega(t)|} \int_{\Omega(t)} s.$$

The coefficient represents the balance between the constant birth rate and the death rate caused by the limited resources. We consider that the resources depend of total amount of population $\frac{1}{|\Omega(t)|}\int_{\Omega(t)} s$ and have a constant distribution in the domain.

We assume that the density of cells into the colony is constant, i.e.,

$$s + d = 1. \tag{1.3}$$

Adding both equations we have that

$$\nabla \cdot v = k_s s - k_d d. \tag{1.4}$$

We take on radial symmetry and therefore the domain $\Omega(t)$ is as follows

$$\Omega(t) = \{x \in \mathbb{R}^3 \text{ so that } |x| < R(t)\}$$

where $R(t)$ is the exterior boundary of the tumor. Velocity "v" of the cells at the boundary determines the expansion of the free boundary, i.e.,

$$\frac{dR}{dt} = v(R(t), t).$$

The aim of the present paper is to prove that:

Theorem 1.1. *For $k_d > 4/3$, there exists a constant steady state*

$$s^* = 1 + \frac{k_d - \sqrt{k_d^2 + 4k_d}}{2},$$

such that, for any positive and regular initial data s_0, the solution s of system (1.2) satisfies

$$\|s - s^*\|_{L^\infty} \to 0, \quad \text{as } t \to \infty$$

and $R(t) \sim R_0\, exp(((1 - s^* + k_d)s^* - k_d)t)$.