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Traveling wave solutions in a diffusive system with two preys and one predator

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ABSTRACT

This paper is concerned with the traveling wave solutions in a diffusive system with two preys and one predator. By constructing upper and lower solutions, the existence of nontrivial traveling wave solutions is established. The asymptotic behavior of traveling wave solutions is also confirmed by combining the asymptotic spreading with the contracting rectangles. Applying the theory of asymptotic spreading, the nonexistence of traveling wave solutions is proved.

1. Introduction

In population dynamics, different predator–prey systems have been proposed to model different processes of energy transmission since Lotka [28] and Volterra [37]. To illustrate and predict some ecological phenomena, much attention has been paid to the dynamics of different predator–prey systems. In particular, to model spatial–temporal behavior of predator and prey, a useful tool is the traveling wave solutions. In the past decades, the existence of traveling wave solutions of two species predator–prey systems has been studied by many researchers, we refer to Dunbar [4–6], Gardner and Smoller [11], Gardner and Jones [10], Gardner [9], Huang et al. [15], Huang [14], Hsu et al. [12], Liang et al. [19], Lin et al. [27], Mischaikow and Reineck [30], Owen and Lewis [32], Wang et al. [38] and references cited therein.

In realistic population communities, there are often several kinds of food and the predator has a choice on which species to feed upon. To model the phenomena, coupled systems containing $n > 2$ equations are needed in population dynamics. Freedman and Waltman [8] established predator-mediated coexistence in a Lotka–Volterra ODE model for two competing species that are preyed upon by a common predator. Cantrell et al. [3] established permanence in the corresponding reaction–diffusion system via the Acyclicity Theorem when the underlying bounded habitat is taken to have an absorbing or a lethal boundary. In this

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paper, we further study the corresponding spatial–temporal patterns by traveling wave solutions. Namely, we shall consider the traveling wave solutions of the following reaction–diffusion system (see Cantrell et al. [3] and Cantrell and Cosner [2])

$$\begin{cases} \dfrac{\partial u_1(x,t)}{\partial t} = d_1\Delta u_1(x,t) + u_1(x,t)\big[a_1 - u_1(x,t) - c_{12}u_2(x,t) - c_{13}u_3(x,t)\big], \\ \dfrac{\partial u_2(x,t)}{\partial t} = d_2\Delta u_2(x,t) + u_2(x,t)\big[a_2 - c_{21}u_1(x,t) - u_2(x,t) - c_{23}u_3(x,t)\big], \\ \dfrac{\partial u_3(x,t)}{\partial t} = d_3\Delta u_3(x,t) + u_3(x,t)\big[a_3 + c_{31}u_1(x,t) + c_{32}u_2(x,t) - u_3(x,t)\big], \end{cases} \tag{1.1}$$

in which $x \in \mathbb{R}$, $t > 0$, and all the parameters except a_3 are positive, here, a_3 may be positive or negative. When $a_3 > 0$, the predator species is a generalist with resources other than the species u_1 and u_2 in (1.1); while $a_3 < 0$, the predator species has no other resources besides the species u_1 and u_2 in (1.1) (see Cantrell et al. [3] and Cantrell and Cosner [2]).

In what follows, we assume that $a_3 > 0$, then (1.1) equals to the following system

$$\begin{cases} \dfrac{\partial u_1(x,t)}{\partial t} = d_1\Delta u_1(x,t) + r_1u_1(x,t)\big[1 - u_1(x,t) - a_{12}u_2(x,t) - a_{13}u_3(x,t)\big], \\ \dfrac{\partial u_2(x,t)}{\partial t} = d_2\Delta u_2(x,t) + r_2u_2(x,t)\big[1 - a_{21}u_1(x,t) - u_2(x,t) - a_{23}u_3(x,t)\big], \\ \dfrac{\partial u_3(x,t)}{\partial t} = d_3\Delta u_3(x,t) + r_3u_3(x,t)\big[1 + a_{31}u_1(x,t) + a_{32}u_2(x,t) - u_3(x,t)\big], \end{cases} \tag{1.2}$$

in which

$$r_i > 0, \qquad d_i > 0, \qquad a_{ij} \geqslant 0, \quad i,j = 1,2,3, \ i \neq j.$$

It is obvious that (1.2) has a trivial steady state $\mathbf{0} = (0,0,0)$, and when

$$a_{12} + a_{13}(1 + a_{31} + a_{32}) < 1, \qquad a_{21} + a_{23}(1 + a_{31} + a_{32}) < 1, \tag{1.3}$$

(1.2) has a unique positive steady state $\mathbf{K} = (k_1,k_2,k_3)$ defined by

$$\begin{aligned} k_1 &= \frac{a_{13}a_{32} - a_{12}a_{23} - a_{23}a_{32} + a_{12} + a_{13} - 1}{a_{12}a_{23}a_{31} + a_{13}a_{32}a_{21} + a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32} - 1}, \\ k_2 &= \frac{a_{23}a_{31} - a_{21}a_{13} - a_{13}a_{31} + a_{21} + a_{23} - 1}{a_{12}a_{23}a_{31} + a_{13}a_{32}a_{21} + a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32} - 1}, \\ k_3 &= \frac{a_{12}a_{21} + a_{31}a_{12} + a_{32}a_{21} - a_{31} - a_{32} - 1}{a_{12}a_{23}a_{31} + a_{13}a_{32}a_{21} + a_{12}a_{21} - a_{13}a_{31} - a_{23}a_{32} - 1}. \end{aligned}$$

For convenience, we first introduce the following definition of traveling wave solutions.

Definition 1.1. A traveling wave solution of (1.2) is a special solution of the form $u(x,t) = \Phi(x + ct)$, where $u = (u_1,u_2,u_3)$ and $\Phi = (\phi_1,\phi_2,\phi_3) \in C^2(\mathbb{R},\mathbb{R}^3)$ is the wave profile that propagates through the one-dimension spatial domain $\mathbb{R}$ at the constant wave speed $c > 0$.

Substituting $u(x,t) = \Phi(x + ct)$ into (1.2) and denoting $x + ct$ as ξ, then (1.2) has a traveling wave solution Φ if and only if Φ is the solution of the following system

$$\begin{cases} d_1\phi_1''(\xi) - c\phi_1'(\xi) + r_1\phi_1(\xi)\big[1 - \phi_1(\xi) - a_{12}\phi_2(\xi) - a_{13}\phi_3(\xi)\big] = 0, \\ d_2\phi_2''(\xi) - c\phi_2'(\xi) + r_2\phi_2(\xi)\big[1 - a_{21}\phi_1(\xi) - \phi_2(\xi) - a_{23}\phi_3(\xi)\big] = 0, \\ d_3\phi_3''(\xi) - c\phi_3'(\xi) + r_3\phi_3(\xi)\big[1 + a_{31}\phi_1(\xi) + a_{32}\phi_2(\xi) - \phi_3(\xi)\big] = 0. \end{cases} \tag{1.4}$$