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Vanishing pressure limits of Riemann solutions to the isentropic relativistic Euler system for Chaplygin gas

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ABSTRACT

The Riemann solutions to the isentropic relativistic Euler system for Chaplygin gas with a small parameter are considered. Unlike the polytropic or barotropic gas cases, we find that firstly, as the parameter decreases to a certain critical number, the two-shock solution converges to a delta shock wave solution of the same system. Moreover, as the parameter goes to zero, that is, the pressure vanishes, the solution is nothing but the delta shock wave solution to the zero-pressure relativistic Euler system. Meanwhile, the two-rarefaction wave solution tends to the vacuum solution to the zero-pressure relativistic system, and the solution containing one rarefaction wave and one shock wave tends to the contact discontinuity solution to the zero-pressure relativistic system as pressure vanishes.

1. Introduction

The isentropic relativistic Euler system reads [29]:

$$\begin{cases} \left(\dfrac{n}{\sqrt{1-v^2/c^2}}\right)_t + \left(\dfrac{nv}{\sqrt{1-v^2/c^2}}\right)_x = 0, \\ \left(\dfrac{(\rho+p/c^2)v}{1-v^2/c^2}\right)_t + \left(\dfrac{(\rho+p/c^2)v^2}{1-v^2/c^2}+p\right)_x = 0, \end{cases} \tag{1.1}$$

which involves conservation laws of baryon numbers and momentum for isentropic perfect fluid in special relativity. Here ρ and n represent the proper mass–energy density and the proper number density of baryons, respectively, $p = p(\rho)$ is the pressure, the constant c is the light speed, v is the particle speed in a chosen Lorenz frame satisfying the relativistic constraint $v^2 < c^2$. Moreover, the sonic speed $\sqrt{p'(\rho)}$ should be not more than the light speed c. The number n is determined by the first law of thermodynamics

$$\theta\, dS = \frac{1}{n}\, d\rho - \frac{\rho + p/c^2}{n^2}\, dn,$$

where θ and S represent the temperature and the entropy per baryon, respectively. Thus, for isentropic fluid we have $dn/n = d\rho/(\rho + p/c^2)$, which yields

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$$n = k_0 \exp\left(\int_1^\rho \frac{ds}{s + p(s)/c^2}\right), \tag{1.2}$$

where k_0 is a constant.

The Riemann problem for (1.1) was solved by Pant in [22] with the equation of state $p = \sigma^2 \rho$, where σ is the sound speed. He also constructed the global weak solution to Cauchy problem for (1.1) under the same equation of state. Li, Shi and Wang [19] generalized the above results with a more general equation of state. We refer to [13,18] for some results about entropy solutions and BV solutions to (1.1). For Chaplygin gas, Cheng and Yang [11] studied (1.1) and obtained the Riemann solutions. In this case, the system is fully linearly degenerate and there appears concentration in solutions when initial data belong to a certain domain in the phase plane. To be more precisely, there are five kinds of solutions, in which four cases contain different discontinuities, while another case involves the so-called delta shock wave. This makes our discussions later more interesting.

The system (1.1) in the Newtonian limit reduces to the classical isentropic Euler equations for compressible fluids:

$$\begin{cases} \rho_t + (\rho v)_x = 0, \\ (\rho v)_t + \left(\rho v^2 + p\right)_x = 0. \end{cases} \tag{1.3}$$

Thus (1.1) can be viewed as the relativistic generalization of (1.3). The Euler system (1.3) is the well-known example of a hyperbolic system of conservation laws and has been widely studied. We can refer to [4] and references cited therein for some known results about the Riemann problem to hyperbolic systems of conservation laws. Specially, Chang and Hsiao [5] considered the Riemann problem and elementary wave interactions of (1.3) for polytropic gas. The solutions involving constant states, rarefaction waves and shock waves were obtained constructively by the phase plane analysis method. For Chaplygin gas, the Riemann solutions to (1.3) were obtained in [2], which involve the concentration phenomenon for certain initial data. That is, the Riemann problem for Chaplygin gas has a unique solution which may include the so-called delta shock wave in some cases. Recently, the Chaplygin gas dynamics has been widely studied and some interesting and important results have been obtained, especially for the Riemann problem. For example, we refer to [9,10,14,16,23,30,31] for the related results and the references cited therein.

As the pressure vanishes, (1.1) formally transforms into the following model

$$\begin{cases} \left(\dfrac{\rho}{\sqrt{c^2 - v^2}}\right)_t + \left(\dfrac{\rho v}{\sqrt{c^2 - v^2}}\right)_x = 0, \\ \left(\dfrac{\rho v}{c^2 - v^2}\right)_t + \left(\dfrac{\rho v^2}{c^2 - v^2}\right)_x = 0, \end{cases} \tag{1.4}$$

called the zero-pressure relativistic Euler system. It can be viewed as a relativistic version of the transport equations

$$\begin{cases} \rho_t + (\rho v)_x = 0, \\ (\rho v)_t + \left(\rho v^2\right)_x = 0, \end{cases} \tag{1.5}$$

which can be used to describe the motion process of free particles sticking under collision in the low temperature and the information of large-scale structures in the universe [12,24]. The transport equations (1.5) have been investigated extensively in the past two decades. The Riemann problem for (1.5) was solved in [1,27], in which the delta shock wave and vacuum do occur. The delta shock wave is characterized by the location, propagation speed, and weight which is the mass of concentrated particles. This shows that the delta shock can be regarded as the galaxies in the universe, or the concentration of particles. We can also see [3,15,21,25,28] for the related results about the delta shock wave.

The method of vanishing pressure limit has been carried out to the Euler system for the isothermal case [17] and for the isentropic case [7,8]. In [17], Li proved that when temperature drops to zero, the solution containing two shock waves converges to the delta shock solution to the transport equations and the solution containing two rarefaction waves converges to the solution involving vacuum to the transport equations. Instead of the isothermal case, Chen and Liu [7] considered the formation of delta shock and a vacuum state of the Riemann solutions to the Euler system in which they took the equation of state as $P = \varepsilon p$ for $p = \rho^\gamma / \gamma$ $(\gamma > 1)$.

In [20], Mitrovic and Nedeljkov considered the generalized pressureless gas dynamics model with a scaled pressure term

$$\begin{cases} \rho_t + \left(\rho g(u)\right)_x = 0, \\ (\rho u)_t + \left(\rho u g(u) + \varepsilon p(\rho)\right)_x = 0, \end{cases} \tag{1.6}$$

where $p = \kappa \rho^\gamma$ for $1 < \gamma < 3$ and g is a non-decreasing function. They extended the results in [7] to (1.6) and found that the delta shock wave appears as the limit of the solutions involving two shock waves as ε goes to zero. Whereafter, Yin and Sheng [32,33] extended the results in [17] and [7] to the Euler system of conservation laws of energy and momentum in special relativity for both isothermal gas and polytropic gas. We also refer to [26] for the related results.