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Active disturbance rejection control based on weighed-moving-average-state-observer ☆

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ABSTRACT

In this work, we are concerned with the boundary stabilization of a one-dimensional anti-stable wave equation corrupted by a boundary disturbance. We firstly propose, by weighed moving average technique, a state observer to make an estimation of the disturbance. Secondly, we design a control by the active disturbance rejection strategy to stabilize the system.

1. Introduction

The observer design has been a major issue in nonlinear system control. It is an important part in the active disturbance rejection control (ADRC) strategy. The main content of the ADRC is composed of three parts, i.e., tracking differentiator, extended state observer and the observer-based closed-loop control. Due to the extended state observer, we are able to make an estimation of the unknown disturbance. Therefore, the disturbance can then be rejected by its estimates. We refer the reader to [1,3,7] for details concerning disturbance rejection problem.

In [3], the backstepping method have been used to deal with the stabilization of wave equation corrupted by uncertainties. It is seen that the backstepping method is very effective for the non-collocated control. For more backstepping controllers or backstepping observers, we refer the reader to [8] and [9] and references therein.

Combining the ADRC method and the backstepping method, Guo [4] give a perfect application to stabilize the one-dimensional anti-stable wave equation. In [4], the disturbance is estimated by a high-gain observer. It is however, due to the large gain constant, there is always a peaking phenomenon associated with the high-gain state observer. In this paper, we first propose a novel state observer which is based on the weighed moving average technique. Then, by using this state observer, we followed the idea of ADRC to stabilize a one-dimensional anti-stable wave equation. Unlike the high-gain observer ours is non-peaking.

Motivated mainly by [4], we are concerned with the stabilization of the following PDEs:

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t), & x \in (0,1),\ t > 0, \\ u_x(0,t) = -q u_t(0,t), & t \geqslant 0, \\ u_x(1,t) = U(t) + d(t), & t \geqslant 0, \end{cases} \tag{1.1}$$

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where u is the state, U is the control, $0 < q \neq 1$ is a constant number and d is unknown disturbance. The system (1.1) represents an anti-stable physical system [7]. The objective of this work is to design an observer-based controller U to stabilize system (1.1) in the presence of disturbance.

We will consider systems (1.1) in the state space $\mathcal{H} = H^1(0,1) \times L^2(0,1)$. The energy of the system is defined by

$$E(t) = \frac{1}{2}\int_0^1 \left|u_t(x,t)\right|^2 dx + \frac{1}{2}\int_0^1 \left|u_x(x,t)\right|^2 dx. \tag{1.2}$$

Then a simple computation shows that

$$\dot{E}(t) = q\left|u_t(0,t)\right|^2 + u_t(1,t)u_x(1,t), \tag{1.3}$$

which shows that system is not dissipative.

Throughout this paper, $v^{(i)}(t)$ represents the i-th order derivative of $v(t)$ at time t. Let $\mathbb{R}$ and $\mathbb{Z}$ be the set of real numbers and the set of integer numbers, respectively. We note

$$\mathbb{R}^+ := \{s \in \mathbb{R} \mid s \geqslant 0\}, \qquad \mathbb{Z}_+ := \{s \in \mathbb{Z} \mid s > 0\} \tag{1.4}$$

and

$$\|f\|_\infty := \sup_{t\in[0,\infty)} \left|f(t)\right|. \tag{1.5}$$

$\mathcal{S}_\lambda[f](t)$ represents the *dilation scaling* of the function $f(t)$, which is defined by

$$\mathcal{S}_\lambda[f](t) := \lambda f(\lambda t), \quad \forall\lambda > 0. \tag{1.6}$$

The convolution used in this paper is given by

$$f_1 * f_2(t) := \int_0^t f_1(s) f_2(t-s)\, ds. \tag{1.7}$$

2. Weighed-moving-average-state-observer

Our state observer considered in this paper is based on a special weight function J, which satisfies

$$\begin{cases} J \in C^\infty(\mathbb{R},\mathbb{R}); \\ \displaystyle\int_0^\infty J(t)\, dt = 1; \\ \operatorname{supp} J^{(i)} \subset (0,1), \quad i = 0,1,2,\ldots. \end{cases} \tag{2.1}$$

Obviously, there are a lot of functions to meet (2.1). For example,

$$J(t) = \begin{cases} C_0 \exp[\frac{1}{t(t-1)}], & t \in (0,1), \\ 0, & \text{else}, \end{cases} \tag{2.2}$$

where C_0 is a positive constant such that

$$\int_0^\infty J(t)\, dt = 1. \tag{2.3}$$

Before giving our main result, we first have a lemma which is very important to the design of the state observer.

Lemma 2.1. *For any $i \in \mathbb{Z}_+$, suppose that the input $v \in C^i(\mathbb{R}^+)$ and $\|v^{(i)}\|_\infty < \infty$. Then, for $\forall t \in [1/\lambda, \infty)$, it follows that*

$$\left|\frac{d^{i-1}}{dt^{i-1}}\left\{\mathcal{S}_\lambda[J]\right\} * v(t) - v^{(i-1)}(t)\right| \leqslant \frac{\|v^{(i)}\|_\infty}{\lambda} \int_0^1 \left|J(\alpha)\right| d\alpha. \tag{2.4}$$