J. Math. Anal. Appl. 417 (2014) 1–22

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

On the well-posedness of higher order viscous Burgers' equations ☆

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ARTICLE INFO

Article history:
Received 28 June 2013
Available online 28 February 2014
Submitted by T. Witelski

Keywords:
Initial value problem
Well-posedness
KdV equation
Dispersive–dissipative models

ABSTRACT

We consider higher order viscous Burgers' equations with generalized nonlinearity and study the associated initial value problems for given data in the L^2-based Sobolev spaces. We introduce appropriate time weighted spaces to derive multilinear estimates and use them in the *contraction mapping principle* argument to prove local well-posedness for data with Sobolev regularity below L^2. We also prove ill-posedness for this type of models and show that the local well-posedness results are sharp in some particular cases viz., when the orders of dissipation p, and nonlinearity $k+1$, satisfy a relation $p = 2k+1$.

1. Introduction

In continuation to our recent works [5,6], here we consider higher order viscous Burgers' equations with generalized nonlinearity which are also known as generalized Korteweg–de Vries (KdV) type equations with dissipative perturbation. These sort of models are well studied in the recent literature, see for example [7,14,19,21] and references therein. The authors in [14,21] considered generalization in the dissipative part, while the authors in [7,19] studied generalization in the nonlinearity. In this work, we are interested in considering generalization in both dissipative as well as nonlinear parts and address the well-posedness issues for the initial value problems (IVPs),

$$\begin{cases} v_t + v_{xxx} + \eta L v + (v^{k+1})_x = 0, \quad x \in \mathbb{R},\ t \geqslant 0,\ k \in \mathbb{N},\ k > 1, \\ v(x,0) = v_0(x), \end{cases} \tag{1.1}$$

☆ The first author is supported by FAPESP grant 2012/23054-6. The second author is supported by grants: FAPESP 2012/20966-4, FAEPEX 1486/12 and CNPq 479558/2013-2.

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http://dx.doi.org/10.1016/j.jmaa.2014.02.056

and

$$\begin{cases} u_t + u_{xxx} + \eta Lu + (u_x)^{k+1} = 0, \quad x \in \mathbb{R},\ t \geqslant 0,\ k \in \mathbb{N},\ k > 1, \\ u(x,0) = u_0(x), \end{cases} \tag{1.2}$$

where $\eta > 0$ is a constant; $u = u(x,t)$, $v = v(x,t)$ are real-valued functions and the linear operator L is defined via the Fourier transform by $\widehat{Lf}(\xi) = -\Phi(\xi)\widehat{f}(\xi)$.

The Fourier symbol $\Phi(\xi)$ is of the form

$$\Phi(\xi) = -|\xi|^p + \Phi_1(\xi), \tag{1.3}$$

where $p \in \mathbb{R}^+$ and $|\Phi_1(\xi)| \leqslant C(1 + |\xi|^q)$ with $0 \leqslant q < p$. The symbol $\Phi(\xi)$ is a real-valued function which is bounded above; i.e., there is a constant C such that $\Phi(\xi) < C$ (see Lemma 2.2 below). We note that, a particular case of $\Phi(\xi)$ in the form

$$\tilde{\Phi}(\xi) = \sum_{j=0}^{n} \sum_{i=0}^{2m} c_{i,j} \xi^i |\xi|^j, \quad c_{i,j} \in \mathbb{R},\ c_{2m,n} = -1, \tag{1.4}$$

with $p := 2m + n$, has been considered in our earlier work [4].

We observe that, if u is a solution of (1.2) then $v = u_x$ is a solution of (1.1) with initial data $v_0 = (u_0)_x$. For this reason Eq. (1.1) is called the derivative equation of (1.2).

As mentioned above, we are interested in studying the well-posedness issues to the IVPs (1.1) and (1.2) for given data in the low regularity Sobolev spaces $H^s(\mathbb{R})$. Recall that, for $s \in \mathbb{R}$, the L^2-based Sobolev spaces $H^s(\mathbb{R})$ are defined by

$$H^s(\mathbb{R}) := \bigl\{ f \in \mathcal{S}'(\mathbb{R}):\ \|f\|_{H^s} < \infty \bigr\},$$

where

$$\|f\|_{H^s} := \bigl\| \langle \xi \rangle^s \widehat{f} \bigr\|_{L^2_\xi},$$

with $\langle \cdot \rangle = 1 + |\cdot|$, and $\widehat{f}(\xi)$ is the usual Fourier transform given by

$$\widehat{f}(\xi) \equiv \mathcal{F}(f)(\xi) := \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-ix\xi} f(x)\, dx.$$

The factor $\frac{1}{\sqrt{2\pi}}$ in the definition of the Fourier transform does not alter our analysis, so we will omit it.

We use the standard notion of well-posedness. More precisely, we say that an IVP for given data in a Banach space X is locally well-posed, if there exist a certain time interval $[0,T]$ and a unique solution depending continuously upon the initial data, and the solution satisfies the persistence property; i.e., the solution describes a continuous curve in X in the time interval $[0,T]$. If the above properties are true for any time interval, we say that the IVP is globally well-posed, and if any one of the above properties fails to hold, we say that the IVP is ill-posed.

In our recent work [5], we considered dissipative perturbation of KdV type equations (i.e., (1.1) and (1.2) with $k = 1$) and proved sharp local well-posedness results for given data with Sobolev regularity below L^2. The IVPs (1.1) and (1.2) with general nonlinearity $k > 0$ are considered in [6] to obtain local well-posedness in H^s, $s \geqslant -1$ and $s \geqslant 0$ respectively.

The sharp local well-posedness results in [5] were obtained by using the *contraction mapping principle* in suitably defined time weighted function spaces. The motivation behind the introduction of time weighted