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Improved blowup results for the Euler and Euler–Poisson equations with repulsive forces

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ABSTRACT

Some results are presented on the formation of singularities in the solutions of the radially-symmetric N-dimensional Euler or Euler–Poisson equations with repulsive forces. Based on the integration method of M.W. Yuen, we generalize the blowup results with constant compact radius R of solutions to the case with general compact radius $R(t)$ and to the case with no compact support restriction.

1. Introduction and main results

In this paper, we prove two blowup results for spherically symmetric solutions to the isentropic Euler ($\delta = 0$) or Euler–Poisson equations with repulsive forces ($\delta = 1$):

$$\begin{cases} \rho_t + \nabla \cdot (\rho \mathbf{u}) = 0, \\ \rho\big[\mathbf{u}_t + (\mathbf{u} \cdot \nabla)\mathbf{u}\big] + \nabla P = \rho \nabla \Phi, \\ \Delta \Phi = \delta \alpha(N) \rho, \end{cases} \tag{1.1}$$

which govern the unknown density $\rho = \rho(t, \mathbf{x}) : \mathbb{R}^+ \times \mathbb{R}^N \mapsto \mathbb{R}^+$ and velocity $\mathbf{u} = \mathbf{u}(t, \mathbf{x}) : \mathbb{R}^+ \times \mathbb{R}^N \mapsto \mathbb{R}^N$ subject to the initial conditions

$$\rho_0(\mathbf{x}) = \rho(0, \mathbf{x}), \qquad \mathbf{u}_0(\mathbf{x}) = \mathbf{u}(0, \mathbf{x}).$$

In the Poisson equation $(1.1)_3$, $\Phi(t, \mathbf{x})$ is the self-gravitational potential field which can be solved as

$$\Phi(t, \mathbf{x}) = \delta \alpha(N) \int_{\mathbb{R}^N} G(\mathbf{x} - \mathbf{y}) \rho(t, \mathbf{y})\, d\mathbf{y},$$

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where G is Green's function for the Poisson equation in the N-dimensional spaces defined by

$$G(\mathbf{x}) := \begin{cases} |\mathbf{x}|, & N = 1, \\ \ln|\mathbf{x}|, & N = 2, \\ -\frac{1}{|\mathbf{x}|^{N-2}}, & N \geqslant 3. \end{cases}$$

Here, $\alpha(N)$ is a constant related to the unit ball in $\mathbb{R}^N$: $\alpha(1) = 1$, $\alpha(2) = 2\pi$ and

$$\alpha(N) = N(N-2)\mathrm{vol}(B_1), \quad N \geqslant 3,$$

where $\mathrm{vol}(B_1) = \frac{\pi^{\frac{N}{2}}}{\Gamma(1+\frac{N}{2})}$ is the volume of the unit ball in $\mathbb{R}^N$. $P = K\rho^\gamma e^S$ is the pressure function which is a common assumption, where S is the specific entropy. If $S = 0$, we say that the system (1.1) is isentropic. If the parameter $K > 0$, we say the system has pressure effects; if $K = 0$, it is pressureless. The constant $\gamma = c_p/c_v \geqslant 1$, where c_p, c_v are the specific heats per unit mass under constant pressure and constant volume respectively, is the ratio of the specific heats, that is, the adiabatic exponent in P. In particular, the fluid is called isothermal if $\gamma = 1$. In this paper, we study system (1.1) in isentropic case.

When $\delta = 0$, (1.1) is the isentropic Euler system. When $\delta = 1$, (1.1) is the Euler–Poisson equations with repulsive forces, such system can be viewed as a semiconductor model, and interested readers can refer to [4,14] for detailed analysis of the system. When $\delta = -1$, (1.1) models fluids that are self-gravitating, such as gaseous stars.

For system (1.1), the results for local existence theories can be found in [15,2,9]. For the results on the construction of analytical solutions for (1.1), see [10,16,7,13,23,24] and the references therein. Moreover, we refer to [6,11,12,25] for theories on the analysis of stabilities of (1.1) and [19] for the dispersive limit of the Euler–Poisson system. Also, we refer to [1,3,5,8,17,18,20,26,27] for the blowup results of the Euler–Poisson system or Euler system and [21,22] for the critical thresholds theory of the Euler–Poisson system.

We are concerned here with the solutions in spherical symmetry

$$\rho(t, \mathbf{x}) = \rho(t, r), \qquad \mathbf{u} = \frac{\mathbf{x}}{r}V(t, r) =: \frac{\mathbf{x}}{r}V, \tag{1.2}$$

where $r = |\mathbf{x}| = (\sum_{i=1}^N x_i^2)^{\frac{1}{2}}$. The Poisson equation $(1.1)_3$ is transformed into

$$r^{N-1}\Phi_{rr} + (N-1)r^{N-2}\Phi_r = \alpha(N)\delta\rho r^{N-1}, \tag{1.3}$$

and by integration on both sides of (1.3), we have

$$\Phi_r = \frac{\alpha(N)\delta}{r^{N-1}} \int_0^r \rho(t, s)s^{N-1}\, ds. \tag{1.4}$$

On the other hand, in the case of spherical symmetry, the equations for ρ and $\mathbf{u}$ in (1.1) can be written in the following form:

$$\begin{cases} \rho_t + V\rho_r + \rho V_r + \dfrac{N-1}{r}\rho V = 0, \\ \rho(V_t + VV_r) + P_r(\rho) = \rho\Phi_r(\rho). \end{cases} \tag{1.5}$$

In [3,5], local blowup conditions are established for pressureless Euler–Poisson equations under some conditions on $\operatorname{div}\mathbf{u}_0$. And in [26], a novel integral condition is obtained to ensure the occurrence of finite time blowup phenomenon for both pressure and pressureless fluids (see also [27]). The main motivation of the paper is to improve the result obtained in [26]. Now, we state the main results of the paper.