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Mindlin–Timoshenko systems with Kelvin–Voigt: analyticity and optimal decay rates

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ABSTRACT

This paper is concerned with asymptotic stability of Mindlin–Timoshenko plates with dissipation of Kelvin–Voigt type on the equations for the rotation angles. We prove that the corresponding evolution semigroup is analytic if a viscoelastic damping is also effective over the equation for the transversal displacements. On the contrary, if the transversal displacement is undamped, we show that the semigroup is neither analytic nor exponentially stable. In addition, in the latter case, we show that the solution decays polynomially and we prove that the decay rate found is optimal.

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1. Introduction

In this paper we study the asymptotic stability of the following Mindlin–Timoshenko plate model

$$\rho h w_{tt} - K\mathcal{L}_1(w,\psi,\varphi) - D_0\Delta w_t = 0 \quad \text{in } \Omega\times\mathbb{R}^+, \tag{1.1}$$

$$\frac{\rho h^3}{12}\psi_{tt} - D\mathcal{L}_2(\psi,\varphi) + K\left(\psi+\frac{\partial w}{\partial x}\right) - D_1\mathcal{L}_2(\psi_t,\varphi_t) = 0 \quad \text{in } \Omega\times\mathbb{R}^+, \tag{1.2}$$

$$\frac{\rho h^3}{12}\varphi_{tt} - D\mathcal{L}_3(\varphi,\psi) + K\left(\varphi+\frac{\partial w}{\partial y}\right) - D_1\mathcal{L}_3(\varphi_t,\psi_t) = 0 \quad \text{in } \Omega\times\mathbb{R}^+, \tag{1.3}$$

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where Ω is a bounded domain of $\mathbb{R}^2$ with Lipschitz boundary $\Gamma = \partial\Omega$, and $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$ are coupling terms defined by

$$\mathcal{L}_1(w,\psi,\varphi) = \frac{\partial}{\partial x}\left(\psi + \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varphi + \frac{\partial w}{\partial y}\right), \tag{1.4}$$

$$\mathcal{L}_2(\psi,\varphi) = \frac{\partial^2\psi}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2\psi}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2\varphi}{\partial x\partial y}, \tag{1.5}$$

$$\mathcal{L}_3(\varphi,\psi) = \frac{\partial^2\varphi}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2\varphi}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2\psi}{\partial x\partial y}. \tag{1.6}$$

This system models the vibrations of a thin plate with reference configuration Ω by taking into account the displacements and rotations caused by the movement. The model was considered in Lagnese [9] and Lagnese and Lions [10] with a comprehensive discussion about its mathematical modeling. Accordingly, the parameters of the model have the following physical meanings. The unknowns w and (ψ,φ) represent respectively, the transverse displacement of the reference surface and the rotation angles of the plate filaments. The constants ρ, h, K, D are positive numbers which represent respectively, the mass density, plate thickness, shear modulus and flexural rigidity. The constant μ is Poisson's ratio which is taken in $(0,1/2)$. The constants D_0, D_1 are nonnegative and related to the presence of damping mechanisms.

The interesting case is when we consider $D_0 = 0$ and $D_1 > 0$, and so we only have damping on the rotation angles ψ and φ. We notice that the damping terms $\mathcal{L}_2(\psi_t,\varphi_t)$ and $\mathcal{L}_3(\varphi_t,\psi_t)$ correspond to the ones of Kelvin–Voigt type. Indeed, materials with Kelvin–Voigt damping are characterized by having stress proportional to strain and strain rate, that is,

$$\sigma = a\varepsilon + b\frac{\partial\varepsilon}{\partial t}, \quad a,b > 0. \tag{1.7}$$

See for instance Bulíček et al. [3]. With respect to Mindlin–Timoshenko models the strain tensor corresponding to rotation equations ψ and φ is given by

$$\varepsilon = \begin{pmatrix} (\frac{\partial\psi}{\partial x} + \mu\frac{\partial\varphi}{\partial y}) & \frac{1-\mu}{2}(\frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}) \\ \frac{1-\mu}{2}(\frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}) & (\frac{\partial\varphi}{\partial x} + \mu\frac{\partial\psi}{\partial y}) \end{pmatrix}. \tag{1.8}$$

See for instance van Rensburg et al. [16]. Then, since the balance of the linear momentum is

$$\rho\frac{\partial^2}{\partial t^2}(\psi,\varphi) = c\nabla\cdot\sigma,$$

where $c > 0$ is a normalizing constant, we get from (1.7)–(1.8),

$$\begin{pmatrix} \rho\psi_{tt} \\ \rho\varphi_{tt} \end{pmatrix} = ac\begin{pmatrix} \frac{\partial^2\psi}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2\psi}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2\varphi}{\partial x\partial y} \\ \frac{\partial^2\varphi}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2\varphi}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2\psi}{\partial x\partial y} \end{pmatrix} + bc\begin{pmatrix} \frac{\partial^2\psi_t}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2\psi_t}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2\varphi_t}{\partial x\partial y} \\ \frac{\partial^2\varphi_t}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2\varphi_t}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2\psi_t}{\partial x\partial y} \end{pmatrix}.$$

But this corresponds precisely to Eqs. (1.2)–(1.3), without coupling terms involving Eq. (1.1).

To the system (1.1)–(1.3) we add initial conditions

$$\begin{aligned} &w(x,y,0) = w_0(x,y), && w_t(x,y,0) = w_1(x,y) \quad \text{in } \Omega, \\ &\psi(x,y,0) = \psi_0(x,y), && \psi_t(x,y,0) = \psi_1(x,y) \quad \text{in } \Omega, \\ &\varphi(x,y,0) = \varphi_0(x,y), && \varphi_t(x,y,0) = \varphi_1(x,y) \quad \text{in } \Omega, \end{aligned} \tag{1.9}$$

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