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An application of bilinear integration to quantum scattering

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ABSTRACT

Scattering theory has its origin in Quantum Mechanics. From the mathematical point of view it can be considered as a part of perturbation theory of self-adjoint operators on the absolutely continuous spectrum. In this work we deal with the passage from the time-dependent formalism to the stationary state scattering theory. This problem involves applying Fubini's Theorem to a spectral measure integral and a Lebesgue integral of functions that take values in spaces of operators. In our approach, we use bilinear integration in a tensor product of spaces of operators with suitable topologies and generalize the results previously stated in the literature.
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1. Introduction

Scattering theory can be considered as a part of the more general perturbation theory in physics [17]. The main idea is that detailed information about an unperturbed self-adjoint operator H_0 (the free Hamiltonian) in a Hilbert space $\mathcal{H}$ enables us to draw conclusions about another self-adjoint operator H (the total Hamiltonian $H = H_0 + V$ where V is the potential) provided that H_0 and H differ little from one to another in an appropriate sense.

There are two main approaches to the mathematical formulation of quantum mechanical scattering theory, the time-dependent and the time-independent or stationary scattering theory.

In the time-dependent scattering theory, we consider the time evolution of an incident particle (wave packet) under the influence of the interaction with a scattering center or with another particle by the evolution equation

$$i\frac{\partial u}{\partial t} = Hu, \quad u(0) = f.$$

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The behaviour of the state u for large times is studied in terms of the free equation $i\partial u_0/\partial t = H_0 u_0$. With the appropriate assumptions on the potential V, for every vector f orthogonal to the eigenvectors of H, there exist vectors $f_0^{(\pm)}$ orthogonal to the eigenvectors of the free Hamiltonian H_0 such that

$$\lim_{t\to\pm\infty}\|u(t)-u_0(t)\| = 0,$$

if $u_0(0) = f_0^{(\pm)}$. The initial data f and $f_0^{(\pm)}$ are related by the equality

$$f = \lim_{t\to\pm\infty} e^{itH}e^{-itH_0}f_0^{(\pm)}$$

because $u(t) = e^{-itH}f$ and $u_0(t) = e^{-itH_0}f_0^{(\pm)}$ for $\pm t \geqslant 0$. The *wave operators*

$$W_\pm = W_\pm(H, H_0) = \lim_{t\to\pm\infty} e^{itH}e^{-itH_0}P_0^{(a)} \tag{1}$$

encode this property provided that the limits in the strong operator topology exist. The operator $P_0^{(a)}$ is the orthogonal projection onto the absolutely continuous subspace $\mathcal{H}_0^{(a)}$ of the operator H_0. The *scattering operator* $S = W_+^*W_-$ connects the asymptotic behaviour of a quantum system as $t\to-\infty$ and $t\to\infty$ in terms of the free problem, that is, $S : f_0^{(-)} \mapsto f_0^{(+)}$.

In the time-independent or stationary scattering theory, one studies solutions of the time-independent Schrödinger equation with a parameter that belongs to the continuous part of the spectrum of the total Hamiltonian operator H. These solutions lie outside the Hilbert space and are characterized by certain asymptotic properties partly motivated by physical considerations. The observables, in particular the S-operator, are obtained from the asymptotic properties of such solutions [4].

It has been well known for a long time that these two methods are mathematically very different. The connections between them has been a problem studied since the seventies. Of fundamental importance is the task to establish conditions for which the final objects of the calculations (the S operator) are identical in both cases. This question is not easy to answer because of the nature of the calculations in the stationary scattering formalism. This theory uses mathematical manipulations that must first be interpreted in some sense before they can be made rigorous so that it is possible to compare with the time-dependent method, which is a very well developed mathematical theory.

The recent developments in scattering theory can be found, for example, in [23] and references therein. Specially important is the work developed by M.Sh. Birman and D.R. Yafaev in stationary scattering theory and for the time-dependent theory, the work developed by Werner O. Amrein, Vladimir Georgescu, J.M. Jauch and K.B. Shina (see for example [2,3,5]). We can cite also the book from Berthier [7] and the work of J. Dereziński and C. Gérard (see for example [10] and references therein).

In this paper we focus our attention in the passage from time-dependent to the stationary formalism. Our starting point is the paper from W.O. Amrein, V. Georgescu, J.M. Jauch [4]. The principal problem to solve in this passage is the following: the basic quantities in the time-dependent theory (e.g. the wave operator) will be expressed in terms of a Bochner integral of certain operators over the time available. These formulas have been known for a long time. Operators in the Bochner integral can be expressed as a spectral integrals via the Spectral Theorem. Then the passage is achieved if we are able to interchange the two integrals and evaluate the time integral. The main problem of a mathematical nature is under which conditions we can interchange the two integrals and verify that the conditions are in fact satisfied for the integrals that we encounter in scattering theory.

In order to develop this alternative approach, we have to change the definition of wave operators $W_\pm$ replacing the unitary groups by the corresponding resolvents $R_0(z) = (H_0 - z)^{-1}$ and $R(z) = (H - z)^{-1}$

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