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Limit cycles for discontinuous quadratic differential systems with two zones

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ABSTRACT

In this paper we study the maximum number of limit cycles given by the averaging theory of first order for discontinuous differential systems, which can bifurcate from the periodic orbits of the quadratic isochronous centers $\dot{x} = -y + x^2$, $\dot{y} = x + xy$ and $\dot{x} = -y + x^2 - y^2$, $\dot{y} = x + 2xy$ when they are perturbed inside the class of all discontinuous quadratic polynomial differential systems with the straight line of discontinuity $y = 0$. Comparing the obtained results for the discontinuous with the results for the continuous quadratic polynomial differential systems, this work shows that the discontinuous systems have at least 3 more limit cycles surrounding the origin than the continuous ones.

1. Introduction

One of the main problems in the qualitative theory of continuous planar polynomial differential systems is the study of their limit cycles, see for instance [13]. The limit cycles of continuous planar quadratic polynomial differential systems has been studied intensively, see for instance the books [9,19] and the hundreds of references quoted therein.

The classification of the quadratic polynomial differential systems having an isochronous center is due to Loud [18]. He proved that after an affine change of variables and a rescaling of the independent variable any quadratic isochronous center can be written as one of the four systems of Table 1.

Chicone and Jacobs proved in [8] that at most 2 limit cycles bifurcate from the periodic orbits of the isochronous center

$$\dot{x} = -y + x^2, \qquad \dot{y} = x + xy, \tag{1}$$

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and that at most 1 limit cycle bifurcate from the isochronous center

$$\dot{x} = -y + x^2 - y^2, \qquad \dot{y} = x + 2xy, \tag{2}$$

when these quadratic centers are perturbed inside the class of all quadratic polynomial differential systems. Their study is based in the displacement function using some results of Bautin [1]. In [4] the authors reproved in an easier way, using the averaging theory, the existence of at least 2 limit cycles bifurcating from the periodic orbits of the center (1) when this is perturbed inside the class of all quadratic polynomial differential systems.

Stimulated by discontinuous phenomena in the real world (see for instance the book [2] and the references quoted therein), a big interest has appeared for studying the limit cycles of discontinuous differential systems, mainly for discontinuous piecewise linear differential systems, see also the paper [15] and the references quoted there.

Our objective is to study the number of limit cycles of the discontinuous quadratic differential systems with two zones separated by a straight line. As far as we know for discontinuous quadratic differential systems only the center problem and the Hopf bifurcation has been studied partially, see [11,10,12]. Related studies about the number of limit cycles bifurcating from center and isochronous centers some of them perturbed in discontinuous quadratic systems can be found in [6,7].

Using the averaging theory of first order we study the maximum number of limit cycles which can bifurcate from the periodic orbits of the isochronous centers (1) and (2) perturbed inside the following class of discontinuous quadratic polynomial differential systems

$$\dot{X}_i = Z_i(x,y) = \begin{cases} Y_1^i(x,y) & \text{if } y > 0, \\ Y_2^i(x,y) & \text{if } y < 0, \end{cases} \tag{3}$$

$i = 1, 2$, where

$$Y_1^1(x,y) = \begin{pmatrix} -y + x^2 + \varepsilon p_1(x,y) \\ x + xy + \varepsilon q_1(x,y) \end{pmatrix},$$

$$Y_2^1(x,y) = \begin{pmatrix} -y + x^2 + \varepsilon p_2(x,y) \\ x + xy + \varepsilon q_2(x,y) \end{pmatrix},$$

$$Y_1^2(x,y) = \begin{pmatrix} -y + x^2 - y^2 + \varepsilon p_1(x,y) \\ x + 2xy + \varepsilon q_1(x,y) \end{pmatrix},$$

$$Y_2^2(x,y) = \begin{pmatrix} -y + x^2 - y^2 + \varepsilon p_2(x,y) \\ x + 2xy + \varepsilon q_2(x,y) \end{pmatrix},$$

ε is a small parameter, and

$$\begin{aligned} p_1(x,y) &= a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2, \\ q_1(x,y) &= b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2, \\ p_2(x,y) &= c_1x + c_2y + c_3xy + c_4x^2 + c_5y^2, \\ q_2(x,y) &= d_1x + d_2y + d_3xy + d_4x^2 + d_5y^2. \end{aligned} \tag{4}$$

In other words, in some sense we extend the work done by Chicone and Jacobs [8] for the continuous quadratic polynomial differential systems to the discontinuous ones with the straight line of discontinuity $y = 0$.